YARN COUNTS AND CALCULATIONS

OXFORD TECHNICAL MANUALS

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PREFACE

One of the first essential or desirable requirements in the preparation and spinning of yarn, as well as in the subsequent operations of cloth structure and weaving, is an adequate knowledge of the subject of Yarn Counts and of Calculations relating thereto.

In the absence of any simplified group of calculations for the principle of counting in regard to Raw Silk, Spun Silk, Artificial Silk, Cotton, Woollen, Worsted, Linen, Hemp and Jute, or even of some unification of counting yarns from the same fibre in every district, it is a distinct advantage to be conversant with a multiplicity of systems.

The present treatise, which appeared originally in serial form in the *Textile Manufacturer* under the *nom de plume* "Themhija," gives consideration to all the above-mentioned fibres, and to yarns made from them, by an elucidation of twenty-two different systems of counting yarns as practised in the various

districts of the United Kingdom, on the Continent, and in America.

The value of the work is that it not only provides the textile student of any branch with practically all that is required to prepare him for his examinations, but also acts as a reference book for spinners, manufacturers, and merchants.

THOMAS WOODHOUSE.

Dundee, 1921.

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CHAPTER I

DEFINITIONS

In all the different branches of the textile industry it has been found necessary to adopt some method of numbering the different sizes of yarn; this invariable custom is practised partly in order that yarns of different thicknesses may be distinguished from each other, and partly for the important and essential function of facilitating calculations when weight has to be taken into account in the process of manufacture.

Although there are many different methods or systems of indicating the number, size, grist, or count of yarn with respect to the relation between its length and its weight, all these systems are modifications of two distinct groups:

I. That in which a given length is constant for any particular system, and the weight of this length is variable.

II. That in which the length is variable and the weight constant for any particular system.

The choice of the length in Group I. or the weight in Group II. is quite arbitrary, and, indeed, there are many variations in both groups. For the sake of distinguishing between the two distinct groups we shall adopt the following definition:

Group I. = fixed-length systems.
Group II. = fixed-weight systems.

In all the fixed-length systems the number or count of the yarn is directly proportional to the sectional area of the yarn; whereas in the fixed-weight systems the number or count of the yarn is inversely proportional to the sectional area of the yarn. In other words, we have

Fixed-length systems: the thicker the yarn, the higher the count.
Fixed-weight systems: the thicker the yarn, the lower the count.

This is the general definition, and although it is not advisable to illustrate the difference in general, an illustration of particular methods conveys this difference quite clearly. Thus, when the unit length (not the fixed length) happens to be the same in two systems, one of which, say jute, is in Group I., and the other, say flax or linen, in Group II., the difference may be demonstrated graphically as in Fig. 1.

The particulars above and below the groups in this figure indicate clearly the relation between the yarns

in the two systems. It must be understood that the two diagrams indicate only a few of the counts in each system. Thus the circles in the upper diagram—the fixed-length system—have diameters equal to 1, 2, 3, 4, 5, 6, 7, and 8 units respectively, and these numbers being the roots of perfect squares, it follows that the circles represent the sectional areas, and therefore the counts, of Nos. 1, 4, 9, 16, 25, 36, 49, and 64. All intermediate counts have been omitted for the sake of simplicity. In a similar manner the circles in the

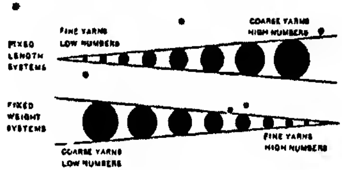


FIG. 1.

lower diagram represent yarns, in the fixed-weight system, the diameters of which are also 8, 7, 6, 5, 4, 3, 2, and 1. The sectional areas of the latter are naturally proportionate to the diameters, but it does not follow that the squares of these eight consecutive numbers represent the counts; as a matter of fact they do not. This phase of the subject will be discussed at a later and more convenient stage.

It is impossible to illustrate satisfactorily all the actual sizes of yarns in one system. The diverging lines in the fixed-length system in Fig. 1, and the

converging lines in the fixed-weight system in the same illustration, are quite pronounced, simply because so many counts between the highest illustrated and the lowest are omitted; in reality, the angle formed by either pair of lines would be exceedingly small, and very much less than those illustrated. In cotton yarns, for example, a thread which is $\frac{1}{16}$ in. in diameter is a very thick yarn, and a very low count; consequently it would be ridiculous to attempt to distinguish the range between the lowest and the highest counts by the method illustrated in Fig. 1.

The diagrams will be referred to later in connection with particular systems, but they also serve to show in general that in all fixed-length systems the yarn receives a higher number or count as the sectional area increases, whereas in the fixed-weight systems the reverse is the case. The fixed-length systems appear on first sight to be the simpler, but it is a significant fact that the fixed-weight systems are the more widely practised.

CHAPTER II

YARN TABLES: SYSTEMS OF COUNTING

THE form in which warp and weft yarns are supplied from the spinning department, or other place, to the preparing and weaving departments depends upon several considerations, but in all cases it is naturally the practice to minimise, as much as is practicable under the existing circumstances, the operations between the spinning department and the preparing and weaving departments. We shall endeavour to indicate the different methods as convenient opportunities arise. For the present it will be sufficient to enumerate the general methods which obtain with regard to what is often termed the hank or loose form.

There is at least one general length unit for each different kind of fibrous material—a length which depends partly upon the size or circumference of the reel upon which the spun yarn is reeled, if it is customary to reel the yarn, partly upon the convenient bulk of the yarn for such unit, and partly upon a convenient relation between the length and the weight. The

reels are in general 1 yd., 1½ yd., 2 yds., or 2½ yds. in circumference, and usually termed respectively 36-in. reel, 54-in. reel, 72-in. reel, and 90-in. reel. There are other sizes of reels used in special industries, but the chief ones are those mentioned. Most branches of the textile industry have their own particular yarn table, the various items of which are multiples of the circumference of the reel used.

COTTON-YARN TABLE

54 in., or one revolution of wrap-reel	= 1½ yd.	= 1 thread.
80 threads	= 120 yds.	= 1 lea.
7 leas	= 840 ,,	= 1 hank.
18 hanks	= 15,120 ,,	= 1 spindle.

The chief item in this table for use in textile calculations is the hank, for which we shall also use the term "length unit" when comparing it with similar chief terms in other tables. The number of these "unit lengths" or hanks (in this particular case) which are required to balance 1 lb. weight indicates the count in the cotton system for the United Kingdom, America, and for many other countries.

It is usual to make up cotton hanks into bundles weighing either 5 lb. or 10 lb. each. Consequently, if it requires 60 hanks to weigh 1 lb., the count of the yarn is 60's, and in a 10 lb. bundle there would be

$$60\text{'s} \times 10 \text{ lb.} = 600 \text{ hanks,}$$

and, naturally, 300 of such hanks in a 5 lb. bundle of the same count. In general,

Count of yarn × weight of bundle
= number of hanks in bundle.

The French or metric system for cotton yarns has a unit length of one thousand metres (one kilometre), and if this length weighs 500 grms. (half-kilogramme) the count of the yarn is 1. Hence there are two metres of No. 1 yarn per gramme, or

$$\frac{2 \text{ metres}}{1 \text{ grm.} \times 2} = 1\text{'s count.}$$

Similarly, if 1000 metres weigh 100 grms., the count is

$$\frac{1000 \text{ metres}}{100 \text{ grms.} \times 2} = 5\text{'s},$$

and generally

$$\frac{\text{Number of metres}}{\text{Weight in grammes} \times 2} = \text{the count.}$$

It might be inferred from the above definitions that there is neither a fixed length nor a fixed weight in this system, but it is really equivalent to a fixed-weight system of ½ kilogramme (500 grms.), because the number of unit lengths or hanks (each 1000 metres) which weigh 500 grms. is equivalent to the count of the cotton yarn.

When this system is compared with that in which the hank is 840 yds., and the fixed weight is 1 lb., we have

	Unit Length.	Fixed Weight.
French	1000 metres	500 grms.
,,	1093·64 yds.	1·1023 lb.
,,	992·144 yds.	1 lb.
United Kingdom	840 yds.	1 lb.

1 metre = 1·0936 yds. 1 kilogramme = 2·2046 lb.

British : French = 840 yds. : 992·144 yds.;

or, in counts, $\frac{\text{British}}{\text{French}} = \frac{992 \cdot 144}{840} = 1 \cdot 181.$

Hence, British count = 1·181 French cotton count, and

$$\text{French cotton count} = \frac{\text{British count}}{1 \cdot 181}.$$

In the United Kingdom the counts of spun silk are reckoned on precisely the same basis as those for cotton for single yarns—*i.e.* the count depends upon the number of hanks which it takes to weigh 1 lb., each hank being 840 yds. in length. (Twisted, folded, doubled, or multiple-ply yarn will be discussed later.)

For raw silk, however, different systems are in use. Warp and weft yarns from this material are often termed respectively "organzine" and "tram."

The Denier System.—The hank or unit length in this system is 400 French ells. The French ell has different values in different parts of France, but the Parisian value is equivalent to 1·19 metres. One metre is equal to 39·37479 in.; hence 400 French ells is equal to

$$400 \text{ French ells} \times 1{\cdot}19 \text{ metres per ell} = 476 \text{ metres.}$$

$$\frac{400 \times 1{\cdot}19 \times 39{\cdot}37479 \text{ in.}}{36 \text{ in. per yard}} = 520{\cdot}569 \text{ yds.}$$

It is usual to reckon 520 yds. for 400 French ells or 476 metres.

The Italian "denier," from which the above denier system derives its name, is a coin equal in weight to 0·001875 of an ounce, and hence there are

$$\frac{1 \text{ ounce}}{0{\cdot}001875} = 533\tfrac{1}{3} \text{ deniers per ounce,}$$

or $$533\tfrac{1}{3} \times 16 = 8533\tfrac{1}{3} \text{ deniers per pound.}$$

The count in the denier system is the weight in deniers, or its equivalent, of 520 yds., or 476 metres, and hence this is also a "fixed-length" system.

$$\text{Weight in deniers of 520 yds.} = \text{the count,}$$

$$\frac{520 \text{ yds.}}{\text{Number of deniers}} = \text{number of yards per denier,}$$

and $$\text{Yards per denier} \times 8533\tfrac{1}{3}$$
$$= \text{number of yards per pound,}$$

i.e. $$\frac{520 \text{ yds.} \times 8533\tfrac{1}{3} \text{ deniers}}{\text{Count}} = \text{yards per pound ;}$$

so that in 40-denier silk yarn there are

$$\frac{520 \times 8533\tfrac{1}{3}}{40 \text{ denier}} = 110933{\cdot}3 \text{ yds. per pound,}$$

and, consequently, equivalent to

$$\frac{110933}{840} = 132\text{'s cotton count or spun-silk count.}$$

The Dram System.—Here the hank or unit length is 1000 yds., and the weight in drams of this hank indicates the count. Thus, if three different hanks weigh severally 2, 3, and 4 drams, the counts are 2's, 3's, and 4's, and so on for all others. This system belongs to the "fixed-length" group.

Yards-per-ounce System.—Here, as in the dram system, the hank is usually one of 1000 yds., and, as indicated by the denomination, the count is determined by the number of yards which weighs 1 oz. For example, if 6 hanks, or 6000 yds., weigh 1 oz., the count is 6000. This system belongs to the "fixed-weight" class.

Woollen warp yarns are spun direct on to the spindles of the mule, or on to some kind of short bobbins or tubes which have been previously dropped on to the spindles; when the yarn is to be made into warps direct the process of reeling is unnecessary. Weft yarns are spun on to bobbins or pirns. Both kinds, however, are sometimes reeled for transportation; they are also reeled for the bulk of yarns which require to be dyed in the hank form. There appear to be at least two distinct sizes of reels: a 72-in. or 2-yd. reel, and a 90-in. or 2½-yd. reel. The former is used extensively in England; whereas the latter finds most favour in Scotland. In some districts in England it is unusual to reel a fixed length for hank-dyeing purposes, the custom being to make reasonably

sized hanks or skeins on a 72-in. reel for handling effectively.

The woollen yarn table in Scotland is very similar to the yarn table used for jute and dry-spun flax yarns. Thus:

SCOTCH WOOLLEN-YARN TABLE

90 in., or 2½ yds. = 1 thread, or the circumference of the reel.
120 threads, or 300 yds. = 1 cut.
12 cuts, or 3600 yds. = 1 slip.
4 slips, or 14,400 yds. = 1 spyndle.

There is a great diversity of woollen systems, many of which, along with other systems employed for other fibrous materials, will appear shortly.

Worsted yarn, which is also made from the wool fibre, was at one time reeled on a 36-in., or 1-yd., reel, but it is now customary to use the same size of reel (54 in.) as is used for cotton and for very fine flax yarns.

WORSTED-YARN TABLE

72 in., or 2 yds. = 1 thread.
40 threads, or 80 yds. = 1 wrap.
7 wraps, or 560 yds. = 1 hank.

WET-SPUN FLAX OR LINEN TABLE

90 in., or 2½ yds. = 1 thread, or the circumference of the reel.

120 threads, or 300 yds. = 1 lea.
10 leas, or 3000 yds. = 1 hank (English).
12 leas, or 3600 yds. = 1 hank (Scotch and Irish).
20 hanks (English) } or 60,000 yds. = 1 bundle.
16⅔ hanks (Scotch or Irish) }

One or more of these bundles are then made into a parcel termed "a lump" or "bunch." No absolutely fixed quantity is adhered to; but, in general, the following may be considered approximately correct:

Up to 14's or 16's . .	1½ bundles per bunch.
16's to 28's	3 ,, ,,
30's to 60's	6 ,, ,,
Above 60's	12 ,, ,,

For the very finest linen yarns, say from 80's or 100's upwards, a 54-in. reel is often used. When this is the case, it is a common practice to lease the yarn in groups of 100 threads or revolutions of the reel. Thus, 100 threads × 54 in. or 1½ yd. = 150 yds., and hence the leasing is in half-leas. The very lowest counts may also be leased in half-leas, and one bundle only made up into a lump or bunch. It must be understood that a wet-spun linen bundle (60,000 yards) is an absolutely constant length, whereas a bundle of jute or dry-spun flax is not an absolutely constant length.

Dry-spun flax or linen yarns and jute yarns are reeled also on the 90-in. reel, but they are made up in a different manner.

JUTE AND DRY-SPUN FLAX TABLE

90 in., or $2\frac{1}{2}$ yds. = 1 thread, or the circumference of the reel.
120 threads, or 300 yds. = 1 cut.
2 cuts, or 600 yds. = 1 heer.
12 cuts / 6 heers, or 3600 yds. = 1 hasp or hank.
4 hanks, or 14,400 yds. = 1 spyndle.

These yarns are made up into bundles weighing as nearly as possible 56 lb. each. Occasionally the bundles weigh a little over 60 lb.

Since much of the jute yarn is thick, it is impracticable to make 12 cut hanks for the heavy sizes; indeed, 12 cut hanks are made only in the finest or lightest jute yarns. A considerable quantity of yarn is made up into what are termed 6-cut hanks. Since 12 cuts form a standard hank, it would be better to distinguish all which differ from this standard length by the name mill hank, for they are made with fewer cuts simply for convenience. Moreover, since most of these yarns are woven in the natural colour, the bulk is not reeled at all. When, however, it is essential to reel the yarn, say for dyeing or other purposes, it is often reeled as under:

Up to 5 lb. yarn .	6 heers	in lease-band		= $\frac{1}{4}$	spyndle.
5 lb. to 10 lb. .	6 cuts	,,	,,	= $\frac{1}{8}$	,,
11 lb. to 24 lb. .	4 cuts	,,	,,	= $\frac{1}{12}$	,,
26 lb. to 36 lb. .	3 cuts	,,	,,	= $\frac{1}{16}$	,,
Heavier yarns .	4 half-cuts	,,	,,	= $\frac{1}{24}$	,,

In addition to the various reels mentioned, which are standard ones, there are several others, the circumferences of which differ from those enumerated. These are used for special cases, particularly in connection with the cord and twine trade; but since the circumferences of these reels are usually for purposes other than that of the count of the yarn, they need not be discussed here.

Each system of counting is, of course, of extreme importance in the particular district or districts in which it is practised; but in the majority of places, or, rather, perhaps, in those places where the largest number of firms are situated, the counts of the yarns spun from the six chief fibres are determined as follows:

Spun silk	by the	number of	hanks of 840	yds.	each in	1 lb.
Cotton	,,	,,	hanks of 840	,,	,,	1 ,,
Woollen	,,	,,	skeins of 256	,,	,,	1 ,,
Worsted	,,	,,	hanks of 560	,,	,,	1 ,,
Linen or flax, wet spun	,,	,,	leas of 300	,,	,,	1 ,,

Dry-spun flax and jute by the weight in pounds of 1 spyndle (14,400 yds.).

Hemp is included sometimes with the wet-spun linen system, but more often with the dry-spun flax and jute system. In all subsequent calculations the units of the length of the first five, and the fixed length of the jute system, will be the ones desired, or

used, unless special reference is made to one or other of the equally important but less widely practised systems.

In practically all yarn calculations used by the English-speaking races the weight of 1 lb. is used either directly or indirectly, so for the purpose of facilitating the conversion or transference of any size of yarn known by a well-defined number or count in one system into the equivalent count of any other system, it will perhaps be the simplest and most satisfactory plan to reduce all the systems—both British and foreign—to the number of yards per pound contained in No. 1 count. It will also simplify matters if all those which belong to the "fixed weight" systems are kept separate from those which belong to the "fixed length" systems. This has been done in Tables I. and II.

[TABLES

TABLE I.—FIXED-WEIGHT SYSTEMS

No.	Name of Fibre.	Denomination.	Unit Length in Yards.	Name of Unit Length.	Fixed Weight.	Yds. per Lb. No. 1, and Conversion Unit.
1	Spun silk .	U.K. and U.S.A.	840	Hank	1 lb.	840
2	Raw silk .	Yards per oz.	1	Yard	1 oz.	16
3	Cotton .	U.K. and U.S.A. cotton	840	Hank	1 lb.	840
4	„ .	French cotton	{1000 metres / 1093·64 yds.}	Hank	{500 grms. / 1·1023 lb.}	992·144
5	Woollen .	Yorkshire skein	1	Skein	1 dram	256
6	„ .	West of England	20	Snap	1 oz.	320
7	„ .	Dewsbury	1	Yard	1 oz.	16
8	„ .	Galashiels	300	Cut	24 oz.	200
9	„ .	Hawick	300	Cut	26 oz.	184·615
10	„ .	Alloa and Stirling	{48 × 240 / 11,520}	{Cuts / Spindle}	24 lb.	480
11	„ .	U.S.A. run	100	Run	1 oz.	1600
12	„ .	U.S.A. cut	300	Cut	1 lb.	300
13	Worsted .	U.K. and U.S.A. worsted	560	Hank	1 lb.	560
14	Linen .	Wet-spun lea	300	Lea	1 lb.	300
15	All yarns .	Metric	1093·64 yds. (1000 metres)	Hank	2·2046 lb. (1000 grms.)	496·072

TABLE II.—FIXED-LENGTH SYSTEMS

No.	Name of Fibre.	Denomination.	Length Unit in Yards.	Name of Length Unit.	Fixed Length in Yards.	Definition of Count in Terms of Weight for Fixed Length.	Number of Yards per Pound.	Con-version Unit.
16	Raw silk . .	Denier	520·569	Hank	520·569	No. of deniers	$\frac{4,437,333}{\text{Count}}$	4,437,333
17	„ . .	Dram	1000	Hank	1000	„ drams	$\frac{1000 \times 256}{\text{Count}}$	256,000
18	Artificial silk .	Denier	520·569	Hank	520·569	„ deniers	$\frac{4,437,333}{\text{Count}}$	4,437,333
19	Woollen . .	Sowerby Bridge	80	..	80	„ drams	$\frac{80 \times 256}{\text{Count}}$	20,480
20	„ . .	Cumberland	..	Bunch	3360	„ ounces	$\frac{3360 \times 16}{\text{Count}}$	53,760
21	„ . .	U.S.A. grain	20	Grain	20	„ grains	$\frac{20 \times 7000}{\text{Count}}$	140,000
22	Jute, dry-spun flax, hemp	*Dundee, or pounds per spyndle	$\left\{\begin{matrix}300\\14,400\end{matrix}\right\}$	$\left\{\begin{matrix}\text{Cut}\\\text{Spyndle}\end{matrix}\right\}$	$\left\{\begin{matrix}14,400\\48 \times 300\end{matrix}\right\}$	„ pounds	$\frac{300 \times 48}{\text{Count}}$	14,400

* Dundee and East of Scotland.

CHAPTER III

CONVERSION OF COUNTS FROM ONE SYSTEM TO ANOTHER SYSTEM

As already inferred, it is often necessary, for various reasons, to convert the count of yarn in one system to the equivalent count in some other system. It is not difficult to make the necessary calculations for any conversion from one system to any other system in either group—*i.e.* in connection with any two systems in the " fixed-weight " group, or in any two systems in the " fixed-length " group—because of the relations demonstrated in Fig. 1. It is essential, however, to exercise more care when converting a count from one system of the fixed-weight group to the equivalent count in one of the systems in the fixed-length group or *vice versa.*

The necessary conversions may be made by calculating from the fundamental principles of counting adopted for the two systems involved, but, as a rule, it will be more easily, and often more correctly, done by one or other of the shorter methods. Never-

theless, for the sake of demonstrating the complete calculations, and to exhibit the actual relations between the various systems, we purpose showing how to change one system to all the others mentioned above. For this object we shall take, as a concrete case, 10's cotton.

In what immediately follows, the reader should note that in the calculations which deal with the fixed-weight systems, the numerator consists of the number of yards per pound in 10's cotton—*i.e.* 10 hanks per pound, or $10 \times 840 = 8400$ yds. per pound. Then the denominator consists of the unit-length in the required system multiplied by the proper ratio to bring it to the number of yards per pound.

Fixed-weight Systems

10's Cotton to Spun Silk

$$\frac{10 \times 840}{840} = 10\text{'s spun silk.}$$

10's Cotton to Raw Silk

$$\frac{10 \times 840}{1 \times 16 \text{ oz. per pound}} = 525 \text{ yds. per ounce raw silk.}$$

10's Cotton to French Cotton

$$\frac{10 \times 840}{1093{\cdot}64 \times \frac{1}{1{\cdot}1023}} = \frac{10 \times 840}{992{\cdot}144} = 8{\cdot}466\text{'s French cotton.}$$

10's Cotton to Yorkshire Skein

$$\frac{10 \times 840}{256} = 32{\cdot}81\text{'s Yorkshire skein.}$$

10's Cotton to West of England

$$\frac{10 \times 840}{1 \times 20 \times 16} = 26{\cdot}25\text{'s West of England.}$$

10's Cotton to Dewsbury Woollen

$$\frac{10 \times 840}{1 \times 16} = 525 \text{ yds. per ounce Dewsbury.}$$

10's Cotton to Galashiels

$$\frac{10 \times 840}{300 \times \frac{16}{24}} = \frac{10 \times 840 \times 24}{300 \times 16} = 42\text{'s Galashiels.}$$

10's Cotton to Hawick

$$\frac{10 \times 840}{300 \times \frac{16}{26}} = \frac{10 \times 840 \times 26}{300 \times 16} = 45{\cdot}5\text{'s Hawick.}$$

10's Cotton to Stirling

$$\frac{10 \times 840}{\frac{48 \times 240}{24}} = \frac{10 \times 840 \times 24}{48 \times 240} = 17{\cdot}5\text{'s Stirling.}$$

10's Cotton to American Run

$$\frac{10 \times 840}{100 \times 1 \times 16} = 5{\cdot}25\text{'s American run.}$$

10's Cotton to American Cut

$$\frac{10 \times 840}{300} = 28\text{'s American cut.}$$

10's Cotton to Worsted

$$\frac{10 \times 840}{560} = 15\text{'s worsted.}$$

10's Cotton to Linen

$$\frac{10 \times 840}{300} = 28\text{'s linen (wet-spun).}$$

10's Cotton to Metric

$$\frac{10 \times 840}{1093{\cdot}64 \times \frac{1}{2{\cdot}2046}} = \frac{10 \times 840}{496{\cdot}072} = 16{\cdot}934\text{'s metric.}$$

FIXED-WEIGHT SYSTEMS TO FIXED-LENGTH SYSTEMS.—The conversion of 10's cotton to the equivalent counts in all the fixed-length systems is performed as under:

10's Cotton to Denier Silk

$$\frac{520{\cdot}569 \times 8533\tfrac{1}{3}}{10 \times 840} = \frac{4437333}{10 \times 840} = 528{\cdot}2\text{'s denier silk.}$$

10's Cotton to Dram Silk

$$\frac{1000 \times 1 \times 256}{10 \times 840} = 30{\cdot}476\text{'s dram silk.}$$

10's Cotton to Sowerby Bridge

$$\frac{80 \times 1 \times 256}{10 \times 840} = 2{\cdot}438\text{'s Sowerby Bridge.}$$

10's Cotton to Cumberland Bunch

$$\frac{3360 \times 16}{10 \times 840} = 6{\cdot}4\text{'s Cumberland bunch.}$$

10's Cotton to American Grain

$$\frac{20 \times 7000}{10 \times 840} = 16{\cdot}6\text{'s American grain.}$$

10's Cotton to Dundee.

$$\frac{14{,}400}{10 \times 840} = 1{\cdot}714\text{'s Dundee (pounds per spyndle).}$$

From the principles demonstrated in these examples it will be an easy matter to convert any other count of cotton to the equivalent count in any of the other systems. Precisely the same principle will apply if it is desired to convert any of the fixed-weight systems to all the others. It will be understood that the conversion of one particular count, or, indeed, of any count, in one system to the equivalent counts in all the other systems will, in virtue of the wide differences in the physical properties of the fibres, and in the systsms of counting, result in some cases in impracticable numbers or counts in certain systems; nevertheless, the principle demonstrated is correct, and may be used in all cases.

To complete the series we might now with advantage convsrt by similar msthods, say, 4-lb. per spyndle jute, or, briefly, 4-lb. yarn, to the remaining systems in the fixed-length group.

Fixed-length Systems

Yards per pound in givsn system
= yards per pound in required system.

$$\therefore \frac{14400}{\text{4-lb. jute}} = 3600 \text{ yds. per lb.}$$

and corresponding counts for this length must be found in the other systems.

4-lb. Jute to U.S.A. Grain

$$\frac{14400 \text{ yds.}}{\text{4-lb. yarn}} = \frac{20 \text{ yds.} \times 7000 \text{ grains}}{\text{U.S.A. grain count}}$$

Hence U.S.A. grain count $= \frac{20 \times 7000 \times 4 \text{ lb.}}{14400} = 38 \cdot 8.$

4-lb. Jute to Cumberland Bunch

$$\frac{14400}{4} = \frac{3360 \text{ yds.} \times 16 \text{ oz. per pound}}{\text{Cumberland bunch}}$$

$$\therefore \frac{3360 \times 16 \times 4}{14400} = 14 \cdot 93\text{'s Cumberland bunch.}$$

4-lb. Jute to Sowerby Bridge

$$\frac{14400}{4} = \frac{80 \text{ yds.} \times 256 \text{ drams per pound}}{\text{Sowerby Bridge}}$$

$$\therefore \frac{80 \times 256 \times 4}{14400} = 5 \cdot 69\text{'s Sowerby Bridge.}$$

4-lb. Jute to Dram Silk

$$\frac{14400}{4} = \frac{1000 \text{ yds.} \times 256 \text{ drams per pound}}{\text{Dram silk}}$$

$$\therefore \frac{1000 \times 256 \times 4}{14400} = 71 \cdot 1\text{'s dram silk.}$$

4-lb. Jute to Denier Silk

$$\frac{14400}{4} = \frac{520 \cdot 569 \times 8533\frac{1}{3} \text{ deniers per pound}}{\text{Silk denier}}$$

$$\therefore \frac{4437333 \times 4}{14400} = 1233\text{'s denier silk.}$$

Since 4-lb. jute yarn is approximately the smallest jute yarn spun—occasionally 3-lb. and $2\frac{1}{2}$-lb. are spun—and is equivalent to such a high count in denier silk, it follows that the highest count in jute, say 450-lb.

rove, and, indeed, practically all other counts, would have no corresponding practicable count in the silk denier scale; hence, it appears unwise to attempt to introduce a scheme of counting which would be uniform for all fibres and yarns. Notwithstanding this obvious drawback to the possibility of including all under one general system, there are sufficient and reasonable grounds for some simplification and unification amongst the various kinds of fibre. But when the lightest yarn in one system is heavier than the heaviest yarn in another system, some doubts arise as to the practicability of embodying the two kinds in the same scheme of counting.

To return to the subject of conversion of the various systems, we have, in general, the following:

Fixed-weight Systems (see Table I., p. 16)

$$\text{(A)} \quad \frac{\text{Given count of yarn} \times \text{conversion unit of given system}}{\text{Conversion unit of required system}} = \text{the required count.}$$

Fixed-length Systems (see Table II., p. 17)

$$\text{(B)} \quad \frac{\text{Given count of yarn} \times \text{conversion unit of required system}}{\text{Conversion unit of given system}} = \text{the required count.}$$

Fixed-weight Systems to Fixed-length Systems (Tables I. and II.)

$$\text{(C)} \quad \frac{\text{Conversion unit of required system}}{\text{Given count of yarn} \times \text{conversion unit of given system}} = \text{the required count.}$$

FIXED-LENGTH SYSTEMS TO FIXED-WEIGHT SYSTEMS
(see Tables I. and II.)

$$\text{(D)} \quad \frac{\text{Conversion unit of given system}}{\text{Given count of yarn} \times \text{conversion unit of required system}} = \text{the required count.}$$

In all cases where the fixed lengths are a multiple or a measure of the length-units in the fixed-weight systems, or *vice versa*, shorter methods of calculation are possible; these valuable practical methods will be pointed out as occasions arise.

In all calculations, except perhaps those which are displayed to illustrate a principle, it is advantageous to eliminate all common factors—in other words, to construct constant values or numbers for the purpose of reducing the amount of figuring and of simplifying the process. The presence of a constant number in certain calculations may be well understood by, say, the employés of one firm, while, on the other hand, the same constant number might be unintelligible to those engaged in another firm. When, however, as in the case of the calculations referring to yarn counts, the various items are known to all and are invariable, the constant numbers derived from these calculations are of general and universal application.

For example, when converting cotton counts to worsted counts, it is clear that both 840 and 560 are divisible by several common factors, the

greatest of which is 280; hence, instead of using the ratio $\frac{840}{560}$, one might as well use the ratio $\frac{3}{2}$.

Again, when converting 4-lb. jute yarn to its equivalent in wet-spun linen or lea count, instead of using all the terms as stated in formula D above and numerically as under,

$$\frac{14400 \text{ yds. per spyndle}}{4 \text{ lb.} \times 300 \text{ yds. per lea}} = 12 \text{ lea yarn},$$

it is much easier, much quicker, and safer to take advantage of the fact that a lea and a cut are the same length, and, since

$$\frac{14400 \text{ yds.}}{300 \text{ yds. per cut or lea}} = 48 \text{ cuts},$$

it is only necessary to use the following,

$$\frac{48 \text{ cuts}}{4 \text{ lb.}} = 12 \text{ lea yarn}.$$

Further, since the ratio $\frac{840}{992 \cdot 144} = 0 \cdot 8466$, the cotton counts might be multiplied by 0·8466 to obtain the equivalent French cotton count, or else the cotton count divided by 1·181, since the ratio $\frac{992 \cdot 144}{840} = 1 \cdot 181$. Other examples, equally simplified, could be adduced.

Perhaps one of the best and simplest methods is to use the conversion units from the last column in

each of the Tables I. and II., pp. 16, 17, and then adopt the name "conversion ratio" for the quotient of any two different conversion units. Thus, if T = the conversion unit of given system, and T^1 = the conversion unit of required system, then

$$\frac{T}{T^1} \text{ and } \frac{T^1}{T} = \text{conversion ratios,}$$

and either used according to circumstances. Further, if G = the given count, and R = the required count, we have the following:

(E) Fixed-weight Systems: $\dfrac{G \times T}{T^1} = R.$

(F) Fixed-length Systems: $\dfrac{G \times T^1}{T} = R.$

Fixed-weight Systems to Fixed-length Systems:

(G) $$\frac{T^1}{G \times T} = R.$$

Fixed-length Systems to Fixed-weight Systems:

(H) $$\frac{T}{G \times T^1} = R.$$

It will be seen that the equations E, F, G, H correspond to those at A, B, C, D (pp. 24, 25).

The necessary matter for the above method is embodied in the three tables of conversion ratios, III., IV., and V., on pp. 30 to 35, in which the values are brought to their lowest terms as fractions, and also to their decimal equivalents. Thus, Table III.

gives all the conversion ratios of the 15 fixed-weight systems; these are shown both vertically and horizontally, and are numbered as in Table I., p. 16. The conversion unit, which is also the number of yards per pound for the systems in Table III., appears in the second vertical column, while the fractions shown opposite any name in a horizontal row of figures indicate the absolute ratios which may be used to convert any count of yarn in this particular system to the corresponding counts in all the other systems above the various ratios. The decimal values immediately under the absolute ratios are sufficiently exact to be used instead of the fractions. The particulars and instructions for each table appear on p. 33.

TABLES

See Instructions for using Tables, p. 33.

TABLE III.—FIXED-WEIGHT SYSTEMS

Count System.	Conversion Unit.	1. Spun Silk.	2. Raw Silk.	3. Cotton.	4. Cotton (French).	5. Woollen (Yorkshire Skein).	6. Woollen (West of England).	7. Woollen (Dewsbury).	8. Woollen (Galashiels).	9. Woollen (Hawick).	10. Woollen (Stirling).	11. Woollen (American Run).	12. Woollen (American Cut).	13. Worsted.	14. Linen (Wet Spun).	15. Metric.
1. Spun Silk	840	1	$\frac{105}{2}$ 52·5	1	$\frac{105}{124}$ 0·8468	$\frac{105}{82}$ 3·2812	$\frac{21}{8}$ 2·625	$\frac{105}{2}$ 52·5	$\frac{21}{5}$ 4·25	$\frac{91}{20}$ 4·55	$\frac{7}{4}$ 1·75	$\frac{21}{40}$ 0·525	$\frac{14}{5}$ 2·8	$\frac{8}{2}$ 1·5	$\frac{14}{5}$ 2·8	$\frac{105}{62}$ 1·6935
2. Raw Silk	16	$\frac{2}{105}$ 0·019	1	$\frac{2}{105}$ 0·019	$\frac{1}{62}$ 0·0161	$\frac{1}{16}$ 0·0625	$\frac{1}{20}$ 0·05	1	$\frac{2}{25}$ 0·08	$\frac{13}{150}$ 0·0867	$\frac{1}{30}$ 0·0334	$\frac{1}{100}$ 0·01	$\frac{4}{75}$ 0·0534	$\frac{1}{35}$ 0·0286	$\frac{4}{75}$ 0·0534	$\frac{1}{31}$ 0·0323
3. Cotton	840	1	$\frac{105}{2}$ 52·5	1	$\frac{105}{124}$ 0·8468	$\frac{105}{32}$ 3·2812	$\frac{21}{8}$ 2·625	$\frac{105}{2}$ 52·5	$\frac{21}{5}$ 4·25	$\frac{91}{20}$ 4·55	$\frac{7}{4}$ 1·75	$\frac{21}{40}$ 0·525	$\frac{14}{5}$ 2·8	$\frac{8}{2}$ 1·5	$\frac{14}{5}$ 2·8	$\frac{105}{62}$ 1·6935
4. Cotton (French)	992	$\frac{124}{105}$ 1·1809	62	$\frac{124}{105}$ 1·1809	1	$\frac{31}{8}$ 3·875	$\frac{31}{10}$ 3·1	62	$\frac{124}{25}$ 4·96	$\frac{403}{75}$ 5·3734	$\frac{31}{15}$ 2·0667	$\frac{31}{50}$ 0·62	$\frac{248}{75}$ 3·3067	$\frac{62}{35}$ 1·7714	$\frac{248}{75}$ 3·3067	2
5. Woollen (Yorkshire Skein)	256	$\frac{32}{105}$ 0·3048	16	$\frac{82}{105}$ 0·3048	$\frac{8}{31}$ 0·2581	1	$\frac{4}{5}$ 0·8	16	$\frac{32}{25}$ 1·28	$\frac{104}{75}$ 1·3867	$\frac{8}{15}$ 0·5334	$\frac{4}{25}$ 0·16	$\frac{64}{75}$ 0·8534	$\frac{16}{35}$ 0·4571	$\frac{64}{75}$ 1·1719	$\frac{16}{31}$ 0·5161
6. Woollen (West of England)	320	$\frac{8}{21}$ 0·381	20	$\frac{8}{21}$ 0·381	$\frac{10}{31}$ 0·3226	$\frac{5}{4}$ 1·25	1	20	$\frac{8}{5}$ 1·6	$\frac{26}{15}$ 1·7334	$\frac{2}{3}$ 0·6667	$\frac{1}{5}$ 0·2	$\frac{16}{15}$ 1·0667	$\frac{4}{7}$ 0·5714	$\frac{16}{15}$ 1·0667	$\frac{20}{31}$ 0·6451
7. Woollen (Dewsbury)	16	$\frac{2}{105}$ 0·019	1	$\frac{2}{105}$ 0·019	$\frac{1}{62}$ 0·0161	$\frac{1}{16}$ 0·0625	$\frac{1}{20}$ 0·05	1	$\frac{2}{25}$ 0·08	$\frac{13}{150}$ 0·0867	$\frac{1}{30}$ 0·0334	$\frac{1}{100}$ 0·01	$\frac{4}{75}$ 0·0534	$\frac{1}{35}$ 0·0286	$\frac{4}{75}$ 0·0534	$\frac{1}{31}$ 0·0323

8. Woollen (Galashiels)	200	5/21 0·2381	25/2 12·5	5/21 0·2381	25/124 0·2016	25/32 0·7812	5/8 0·625	25/2 12·5	1	18/12 1·083	5/12 0·4167	1/8 0·125	2/3 0·6667	5/14 0·8571	2/3 0·6667	25/62 0·4032
9. Woollen (Hawick)	184 8/13 2400/13	20/91 0·2198	150/13 11·5385	20/91 0·2198	75/403 0·1861	75/104 0·7211	15/26 0·5769	150/13 11·5385	12/13 0·9231	1	5/13 0·3846	3/26 0·1154	8/13 0·6154	30/91 0·3297	8/13 0·6154	150/403 0·3722
10. Woollen (Stirling)	480	4/7 0·5714	30	4/7 0·5714	15/31 0·4839	15/8 1·875	3/2 1·5	30	12/5 2·4	13/5 2·6	1	3/10 0·3	8/5 1·6	6/7 0·8571	8/5 1·6	30/31 0·9677
11. Woollen (American Run)	1600	40/21 1·9048	100	40/21 1·9048	50/31 1·6129	25/4 6·25	5	100	8	26/3 8·6667	10/3 3·3334	1	16/3 5·3334	20/7 2·8571	16/3 5·3334	100/31 3·2258
12. Woollen (American Cut)	300	5/14 0·3571	75/4 18·75	5/14 0·3571	75/248 0·3024	75/64 1·1719	15/16 0·9375	75/4 18·75	3/2 1·5	13/8 1·625	5/8 0·625	3/16 0·1875	1	15/28 0·5357	1	75/124 0·6048
13. Worsted	560	2/3 0·6667	35	2/3 0·6667	35/62 0·5645	35/16 2·1875	7/4 1·75	35	14/5 2·8	91/30 3·0334	7/6 1·1667	7/20 0·35	28/15 1·8667	1	28/15 1·8667	35/31 1·129
14. Linen (Wet Spun)	300	5/14 0·3571	75/4 18·75	5/14 0·3571	75/248 0·3024	75/64 1·1719	15/16 0·9375	75/4 18·75	3/2 1·5	13/8 1·625	5/8 0·625	3/16 0·1875	1	15/28 0·5357	1	75/124 0·6048
15. Metric	496	62/105 0·5905	31	62/105 0·5905	1/2 0·5	31/16 1·9375	31/20 1·55	31	62/25 2·48	403/150 2·6867	31/30 1·0334	31/100 0·31	124/75 1·6534	31/35 0·8857	124/75 1·6534	1

TABLE IV.—FIXED-LENGTH SYSTEMS

Count System.	Conversion Unit.	16. Silk, Denier.	17. Silk, Dram.	18. Silk, Artificial.	19. Woollen (Sowerby Bridge).	20. Woollen (Cumberland Bunch).	21. Woollen (U.S.A. Grain).	22. Jute.
		Conversion Ratios.						
16. Silk, Denier	4,437,333	1	$\frac{256,000}{4,437,333}$ 0·0577	1	$\frac{20,480}{4,437,333}$ 0·0046	$\frac{17,920}{1,479,111}$ 0·0121	$\frac{140,000}{4,437,333}$ 0·0316	$\frac{1600}{493,037}$ 0·0032
17. Silk, Dram	256,000	$\frac{4,437,333}{256,000}$ 17·3333	1	$\frac{4,437,333}{256,000}$ 17·3333	$\frac{2}{25}$ 0·08	$\frac{21}{100}$ 0·21	$\frac{35}{64}$ 0·5469	$\frac{9}{160}$ 0·0563
18. Silk, Artificial	4,437,333	1	$\frac{256,000}{4,437,333}$ 0·0577	1	$\frac{20,480}{4,437,333}$ 0·0046	$\frac{17,920}{1,479,111}$ 0·0121	$\frac{140,000}{4,437,333}$ 0·0316	$\frac{1600}{493,037}$ 0·0032
19. Woollen (Sowerby Bridge)	20,480	$\frac{4,437,333}{20,480}$ 216·6666	$\frac{25}{2}$ 12·5	$\frac{4,437,333}{20,480}$ 216·6666	1	$\frac{21}{8}$ 2·625	$\frac{875}{128}$ 6·8359	$\frac{45}{64}$ 0·7031
20. Woollen (Cumberland Bunch)	53,760	$\frac{1,479,111}{17,920}$ 82·5397	$\frac{100}{21}$ 4·7619	$\frac{1,479,111}{17,920}$ 82·5397	$\frac{8}{21}$ 0·381	1	$\frac{875}{336}$ 2·6042	$\frac{15}{56}$ 0·2679
21. Woollen (U.S.A. Grain)	140,000	$\frac{4,437,333}{140,000}$ 31·6952	$\frac{64}{35}$ 1·8286	$\frac{4,437,333}{140,000}$ 31·6952	$\frac{128}{875}$ 0·1463	$\frac{36}{125}$ 0·384	1	$\frac{18}{175}$ 0·1029
22. Jute (British Empire)	14,400	$\frac{493,037}{1600}$ 308·1481	$\frac{160}{9}$ 17·7778	$\frac{493,037}{1600}$ 308·1481	$\frac{64}{45}$ 1·4223	$\frac{56}{15}$ 3·7333	$\frac{175}{18}$ 9·7223	1

INSTRUCTIONS FOR USING TABLES

TABLE III.—To convert counts from systems mentioned in vertical column of names to systems mentioned in horizontal column of names, multiply by the "conversion ratio" opposite the given system in the horizontal row of ratios and under the required system in the vertical row of ratios. These ratios are given in fractional and decimal values, and the latter are carried out to four places when necessary. In general, however, the nearest value to two places of decimals will be sufficient. If absolute values are required the fractional ratio should be used. *Ex.* To convert Yorkshire Skein Woollen to Dewsbury Woollen: move along No. 5 horizontal column until the number in No. 7 vertical column is reached. This is marked by an asterisk. Hence Yorkshire Skein count $\times 16 =$ Dewsbury count.

TABLE IV.—To convert a count in one Fixed-length System to the equivalent count in another Fixed-length System: multiply the given count by the "conversion ratio," and the result is the count required. Given systems in vertical column of names: required systems in horizontal column of names. (Method exactly as in Table III.)

TABLE V.—Except under very exceptional circumstances, it is unnecessary to use the absolute values shown by the fractions. In the above table the absolute values are converted to whole numbers and decimals.

The "conversion ratios" in this table can be used for Fixed-weight Systems to Fixed-length Systems, and for Fixed-length Systems to Fixed-weight Systems—*i.e.* horizontal to vertical and vertical to horizontal. Dividing the "conversion ratio" by the given count in any system gives the equivalent count in the required system. When the given system is in the vertical column of names the conversion ratio is $\frac{T^1}{T}$ (see equations (C) and (G), pp. 24 and 27), and when the given system is in the horizontal row of names the same conversion ratio clearly becomes $\frac{T}{T^1}$ (see equations (D) and (H), pp. 25 and 27).

TABLE V.—FIXED WEIGHT TO FIXED LENGTH, AND FIXED LENGTH TO FIXED WEIGHT

Count System.	Conversion Unit.	4,437,333	256,000	4,437,333	20,480	53,760	140,000	14,400
		16. Silk, Denier.	17. Silk, Dram.	18. Silk, Artificial.	19. Woollen (Sowerby Bridge).	20. Woollen (Cumberland Bunch).	21. Woollen (U.S.A. Grain).	22. Jute (British Empire).
		Conversion Ratios.						
1. Spun Silk	840	$\frac{1,479,111}{280}$ 5282·5	$\frac{6400}{21}$ 304·8	$\frac{1,479,111}{280}$ 5282·5	$\frac{512}{21}$ 24·38	64	$\frac{500}{3}$ 166·7	$\frac{120}{7}$ 17·14
2. Raw Silk	16	$\frac{4,437,333}{16}$ 277,333	16,000	$\frac{4,437,333}{16}$ 277,333	1280	3360	8750	900
3. Cotton	840	$\frac{1,479,111}{280}$ 5283	$\frac{6400}{21}$ 304·8	$\frac{1,479,111}{280}$ 5283	$\frac{512}{21}$ 24·38	64	$\frac{500}{8}$ 166·7	$\frac{120}{7}$ 17·14
4. Cotton (French)	992	$\frac{4,437,333}{992}$ 4473	$\frac{8000}{31}$ 258·1	$\frac{4,437,333}{992}$ 4473	$\frac{640}{31}$ 20·65	$\frac{1680}{31}$ 54·19	$\frac{4375}{31}$ 141·1	$\frac{450}{31}$ 14·52
5. Woollen (Yorkshire)	256	$\frac{4,437,333}{256}$ 17,333	1000	$\frac{4,437,833}{256}$ 17,333	80	210	$\frac{4375}{8}$ 546·9	$\frac{225}{4}$ 56·25
6. Woollen (W. of England)	320	$\frac{4,437,833}{320}$ 13,867	800	$\frac{4,437,333}{820}$ 13,867	64	168	$\frac{875}{2}$ 437·5	45

7. Woollen (Dewsbury)	16	$\frac{4,437,333}{16}$ 277,333	16,000	$\frac{4,437,333}{16}$ 277,333	1280	8360	8750	900
8. Woollen (Galashiels)	200	$\frac{4,437,333}{200}$ 22,187	1280	$\frac{4,437,333}{200}$ 22,187	$\frac{512}{5}$ 102·4	$\frac{1344}{5}$ 268·8	700	72
9. Woollen (Hawick)	$184\frac{4}{15}$	$\frac{19,228,443}{800}$ 24,036	$\frac{4160}{3}$ 1387	$\frac{19,228,443}{800}$ 24,036	$\frac{[illegible]}{15}$ [illegible]0·9	$\frac{1456}{5}$ 291·2	$\frac{2275}{3}$ 758·3	78
10. Woollen (Stirling)	480	$\frac{1,479,111}{160}$ 9244	$\frac{1600}{3}$ 533·3	$\frac{1,479,111}{160}$ 9244	$\frac{128}{3}$ 42·67	112	$\frac{875}{3}$ 291·7	80
11. Woollen (U.S.A. Run)	1600	$\frac{4,437,338}{1600}$ 2773	160	$\frac{4,437,333}{1600}$ 2773	$\frac{64}{5}$ 12·8	$\frac{168}{5}$ 33·6	$\frac{175}{2}$ 87·5	9
12. Woollen (U.S.A. Cut)	300	$\frac{1,479,111}{[illegible]}$ 14,791	$\frac{2560}{3}$ 853·3	$\frac{1,479,111}{100}$ 14,791	$\frac{1024}{15}$ 68·26	$\frac{896}{5}$ 179·2	$\frac{1400}{3}$ 466·7	48
13. Worsted	560	$\frac{4,437,338}{560}$ 7924	$\frac{3200}{7}$ 457·1	$\frac{4,437,333}{560}$ 7924	$\frac{256}{7}$ 3[illegible]·57	96	250	$\frac{180}{7}$ 25·71
14. Linen (Wet Spun)	300	$\frac{1,479,111}{100}$ 14,791	$\frac{2560}{3}$ 853·8	$\frac{1,479,111}{100}$ 14,791	$\frac{1024}{15}$ 68·26	$\frac{896}{5}$ 179·2	$\frac{1400}{3}$ 466·7	48
15. Metric	496	$\frac{4,437,333}{496}$ 8946	$\frac{16,000}{31}$ 516·1	$\frac{4,437,383}{496}$ 8946	$\frac{1280}{31}$ 41·29	$\frac{8360}{31}$ 108·4	$\frac{8750}{31}$ 282·3	$\frac{900}{31}$ 29·03

CHAPTER IV

MULTIPLE-PLY YARNS: SHRINKAGE NEGLECTED

When two or more single yarns or threads are joined, or rather twisted together, more or less intimately, the resulting compound structure is termed 2-fold or 2-ply, 3-fold or 3-ply, etc. The direction of the twist in the compound thread is almost invariably in the opposite direction to the twist of the single threads from which it is formed. Similarly, if two or more of these compound threads are again twisted to form superior and level compound threads, as in the case of several yarns for twines, cords, ropes, fancy jute carpet warp yarns, and the like, the direction of the twist is again reversed—that is, the twist is then in the same direction as that of all the single threads, and the finished product is then termed "cable laid" or cable yarn. Each additional yarn or thread employed in the formation of compound threads naturally increases the thickness of the latter, and, with the single exception of silk, such addition is accompanied by an alteration in the count.

The counts of all multiple-twisted threads, with the above exception of silk, in both the fixed-weight systems and the fixed-length systems, are represented symbolically by two or more numbers, or rather groups of numbers, usually two. When the compound thread can be represented by two groups of numbers, the first group indicates the number of individual threads of which the compound thread is made, and the second group indicates in some way or other the count of the individual threads. Thus, in the simplest form, the symbol 2/10's shows that two single threads, each of 10's count in any system except silk, are twisted together. Similarly, if 3, 4, 5 . . . n threads are twisted together, the compound yarns would be indicated by 3/10's, 4/10's, 5/10's . . . n/10's. It need hardly be stated that the count of the compound thread is not 10's but some other number which will be explained almost immediately. Further, if, say, two of these compound threads are twisted together to form a cable cord, the symbol used would be 2/2/10's, 2/3/10's . . . 2/n/10's.

On the other hand, the symbols for twisted, folded, or multiple-ply silk yarns contain in the first group the actual count of the yarn, and in the second group the number of threads which are folded to make the count of the compound structure. For example, if 2, 3, 4 . . . n threads are twisted together (say, 8400 yds., or 10 hanks of 840 yds. each), all of these

different threads in the compound state weigh 1 lb., then the count is equivalent to 10's silk, and the symbols would be 10's/2, 10's/3, 10's/4 . . . 10's/*n*.

It will thus be seen that since the count of the multiple-ply silk yarns is determined by the number of hanks per lb., as indicated by the count number which precedes the number of threads in the symbol, and as if the compound thread were a single thread of the count mentioned, the actual thickness of the individual threads decreases as the number of combined threads increases. For example, although 10/2, 10/3 . . . 10/*n* means in each case the equivalent of 10's single so far as the compound count goes, the individual threads which form the 10/2 are thicker than the individual threads which form the 10/3, while those of the 10/3 are, in turn, thicker than those of the 10/4, and so on.

When all the individual threads which form a compound thread are of the same thickness or count, the value, number, or count of the compound thread, neglecting the amount of take-up or shrinkage due to the process of twisting, is found perhaps equally well in the two distinct groups of systems. When, however, a multiple thread consists of individual threads of different thicknesses, the resulting compound count is, in general, more easily found in the fixed-length systems than in the fixed-weight systems; for in all cases the count of the compound yarn in the

fixed-length systems—when the shrinkage is neglected—is the aggregate weight of the yarns so combined. Hence, if 2, 3, 4 . . . n threads, all of the same size, from any one of the fixed-length systems in Table II., p. 17, are twisted together, the count is simply 2, 3, 4 . . . n times the count of one of the individual threads, plus, of course, the usually slight addition due to contraction of some or all of the threads in twisting. Similarly, and with the necessary allowance for contraction, if all the threads which are twisted together are of different counts, the resulting count of the compound yarn is the sum of the individual counts. The latter example is, of course, more or less hypothetical, for in the majority of cases a compound yarn consists of a number of individual yarns of the same count. Nevertheless, the method is obviously applicable in all cases. Both examples are evidently included in the following statement:

Fixed-length Systems.—The count of any compound yarn, made from two or more individual threads by the process of twisting, is the sum of the counts of the two or more individual threads, when the contraction or take-up due to the operation of twisting the threads is neglected.

Example: If 8 threads of 5-lb. jute or dry-spun flax are twisted together, the count of the compound yarn is

$$5+5+5+5+5+5+5+5=40 \text{ lb.}$$

or simply

$$5 \text{ lb.} \times 8 \text{ ply} = 40 \text{ lb.}$$

If numbers only are used, *i.e.* the name lb. eliminated, the example will do for any of the other systems in the fixed-length group.

A different method must be followed in connection with the count of compound yarns in the fixed-weight systems. If, however, one keeps in mind the important, though simple, fact that any number of unit-lengths (hanks, leas, cuts, skeins, etc.) divided by the weight in pounds, gives the count, the calculations are generally easy. Thus, suppose that 2 threads of 20's are to be twisted together, and for the sake of simplicity we take, for calculation purposes, 20 hanks of each of the two yarns, or 40 hanks in all, then, since the 40 hanks weigh 2 lb., and since when the two are twisted together, there will be, neglecting contraction, 20 hanks of twist weighing 2 lb.

$$\therefore \frac{20 \text{ hanks of twist}}{2 \text{ lb. of material}} = 10 \text{ hanks per pound.}$$

Therefore, according to definition, the 2/20's yarn is equivalent to 10's single. Similarly, if 20 hanks of each of 4 threads of 20's were twisted together, there would be

$$\frac{20 \text{ hanks of twist}}{4 \text{ lb. of material}} = 5 \text{ hanks per pound.}$$

Hence, 4/20's is equal in count to 5's single. In

general, to find the count of a compound thread in any fixed-weight system, when all the individual threads are of the same size, and when contraction due to twisting is neglected, it is sufficient to divide the count of the single yarn employed by the number of threads which are twisted together.

In the manufacture of fancy twist yarns there may be, and often are, not only two or more individual threads twisted together, but the individual yarns are of different count. Moreover, it is not uncommon to find two or more different materials, also differing in thickness and in the lengths required, combined for some specific purpose. It is in such cases as these that the greatest amount of care and observation should be practised, not so much, perhaps, to find the resulting count of the compound yarn—although this is often necessary or desirable in regard to the weight of the compound yarn, if much of it is used —as to estimate its value correctly. Consequently, it is essential and desirable to be able to find the resulting count of a compound structure when any number of threads of the same material, or of different materials, are twisted together.

Consider first the case where all the threads which are united come under the same system of counting, and where the take-up in the operation of twisting is neglected. Say, two threads of any of the materials, and from any of the fixed-weight systems in Table I.,

p. 16, are twisted together, and the counts of these two individual threads are 24's and 12's. Then, from the definition of the counts, it is clear that

1 hank, lea, cut, or skein of 24's weighs						$\frac{1}{24}$ lb.
1	„	„	„	12's	„	$\frac{1}{12}$ „
2 hanks					weigh	$\frac{1}{24}+\frac{1}{12}$ lb.

Therefore, when these two hanks of different thicknesses are compounded by twisting, there will be one hank of twist weighing $\frac{1}{24}+\frac{1}{12}=\frac{1}{8}$ lb.

Now, as already mentioned, any number of hanks, leas, cuts, or skeins divided by the weight in pounds of such number equals the count, hence

$$\frac{1 \text{ hank}}{\frac{1}{8} \text{ lb.}}=\frac{1\times 8}{1}=8\text{'s count}$$

in any of the fixed-weight systems.

Although the above method of taking the weight of one hank of each thread is quite correct in principle for any number of threads which are twisted together, it is much simpler in general to take a larger number of hanks for the sake of facilitating the calculation. Probably the most satisfactory number is that represented by the least common multiple of the various threads which are to be combined, but since the operation of finding the L.C.M. might in many cases increase instead of decrease the length of the arithmetical process, it is a common practice, and a very good practice, to consider as many hanks, leas, cuts,

or skeins of each individual thread as are represented by the number of hanks per pound of the highest count in the combination. Thus, in the above example, 24's is higher than 12's, and hence 24 hanks of each yarn might be taken in the calculation.

24 hanks	÷ 24's yarn	= 1 lb.
24 ,,	÷ 12's ,,	= 2 ,,
48 hanks		3 lb.

48 hanks of single yarn or 24 hanks of twist yarn weigh 3 lb., therefore

$$\frac{24 \text{ hanks of twist}}{3 \text{ lb. of material}} = 8\text{'s count, as before.}$$

Again, say three threads are twisted together: a 24's, a 20's, and a 12's. Then

$$\begin{aligned} 24 \div 24 &= 1 \\ 24 \div 20 &= 1\tfrac{1}{5} \\ 24 \div 12 &= 2 \\ \hline & \ 4\tfrac{1}{5} \text{ lb. in 24 hanks of twist.} \end{aligned}$$

$$\therefore \frac{24}{4\frac{1}{5}} = \frac{24 \times 5}{21} = 5\tfrac{5}{7}\text{'s count.}$$

To take the least-common-multiple method, we find that the L.C.M. of 24, 20, and 12 is 120. Hence

120 ÷ 24 =	5 lb.
120 ÷ 20 =	6 ,,
120 ÷ 12 =	10 ,,
	21 lb.

and

$$\frac{120 \text{ hanks of twist}}{21 \text{ lb. of material}} = 5\tfrac{5}{7}\text{'s count.}$$

When two threads only are twisted together, and this number for ordinary weaving yarns forms the bulk, the resulting count of the compound yarn is often found as follows:

$$\frac{\text{The product of the two counts}}{\text{The sum of the two counts}} = \text{the resulting count.}$$

Using the same two counts as in the first example to demonstrate this method, we have

$$\frac{24 \times 12}{24 + 12} = \frac{288}{36} = 8\text{'s, as before.}$$

The above method is simply a particular case of the general principle, in which the number of hanks is represented, not by the highest count but by the product of the two counts; if the yarns are represented by the letters A and B, it is evident by the general method that

$$\begin{array}{rl} AB \div A = & B \text{ lb.} \\ AB \div B = & A \text{ lb.} \\ \hline & A + B \text{ lb.} \end{array}$$

$$\therefore \frac{(A \times B) \text{ hanks}}{(A + B) \text{ lb.}} = C\text{, the resulting count.}$$

This formula is true in all cases, hence

$$\begin{aligned} AB &= (A+B)C \\ &= AC + BC \\ AB - AC &= BC \\ A(B-C) &= BC \\ \therefore\ A = \frac{BC}{B-C} &\text{ or } B = \frac{AC}{A-C}. \end{aligned}$$

Consequently, if A and B are two threads which, when twisted together, have a count equal to C, it will be seen that the count C of the twist yarn can be obtained in several ways. It is only necessary to choose one thread—say A or B—and to find the other by one of the formulæ given. Such operations as the latter are desirable when the count of a compound thread should be a whole number.

Example: Find what count of yarn should be twisted with a 24's yarn to produce a twist yarn equal to 8's count.

$$A = \frac{24 \times 8}{24 - 8} = \frac{192}{16}$$
$$= 12\text{'s}.$$

Or, again, what count of yarn should be used in conjunction with a 16's count for a twist yarn equal to 8's count?

$$A = \frac{16 \times 8}{16 - 8} = \frac{128}{8}$$
$$= 16\text{'s}.$$

This latter is, of course, obvious from a consultation

of the first definition of twisting yarns of the same count.

The same principle of finding the resulting count of a compound thread is applicable to any number of threads, although, as already indicated, it is only necessary to adopt this particular method in practice when great accuracy is desired in the resulting count of the compound yarn. The highest count divided by all the others in succession is sufficiently accurate in most cases. In order to complete the scheme, however, we might give the formula to be used for three threads, and finally the general formula of the assemblage of any number of threads.

Let P, Q, and R represent the counts of three threads which are to be twisted together. Then $P \times Q \times R =$ the product of the counts, and also the number of hanks to be considered in the calculation.

$$\begin{aligned} PQR \div P &= \quad QR \text{ lb.} \\ PQR \div Q &= \quad PR \text{ lb.} \\ PQR \div R &= \quad \underline{PQ \text{ lb.}} \\ & \quad (QR + PR + PQ) \text{ lb.} \end{aligned}$$

Hence

$$\frac{PQR}{PQ + QR + RP} = C, \text{ the resulting count.}$$

If it were required to find the count of a thread R which, when twisted with two other known counts P and Q, would result in a compound thread of count C, then

$$\begin{aligned}
PQR &= C(PQ+QR+RP)\\
&= CPQ+CQR+CRP,\\
PQR-CQR-CRP &= CPQ,\\
R(PQ-CQ-CP) &= CPQ.\\
\therefore\ R &= \frac{CPQ}{PQ-CQ-CP}.
\end{aligned}$$

Thus, to take a simple combination, if P = 40's, Q = 20's, and C = 5's, find R.

$$\begin{aligned}
R &= \frac{5\times 40\times 20}{(40\times 20)-(5\times 20)-(5\times 40)}\\
&= \frac{4000}{800-100-200} = \frac{4000}{800-(100+200)}\\
&= \frac{4000}{500} = 8\text{'s}.
\end{aligned}$$

In order to prove this result, we will find the resulting count of a compound thread made up of 40's, 20's, and 8's by the usual method, which is almost invariably the simplest for this type of calculation.

40 hanks	÷ 40's =	1 lb.
40 ,,	÷ 20's =	2 lb.
40 ,,	÷ 8's =	5 lb.
120 hanks		8 lb. for 40 hanks of twist.

$$\therefore\ \frac{\text{40 hanks of twist}}{\text{8 lb. of material}} = 5\text{'s, the resulting count.}$$

An even simpler method of finding the 8's yarn—the unknown count—in the second last example displayed is as under:

Take a number of hanks which corresponds to the counts of the highest—*i.e.* the hanks per pound of the highest count. Then

$$\frac{40 \text{ hanks of twist}}{5\text{'s twisted count}} = 8 \text{ lb. weight.}$$

$$\begin{aligned} 40 \text{ hanks} \div 40\text{'s} &= 1 \text{ lb.} \\ 40 \quad ,, \quad \div 20\text{'s} &= 2 \text{ lb.} \\ \hline & \; 3 \text{ lb.} \end{aligned}$$

Consequently, the unknown count is

$$\frac{40 \text{ hanks}}{8 \text{ lb.} - 3 \text{ lb.}} = \frac{40}{5} = 8\text{'s, as before;}$$

and this simple method is obviously applicable in the case of two or of any number of threads.

These calculations may be done in stages, if desired, but, as a rule, it is better to adopt the methods demonstrated.

The general formula where 1,2,3, . . . n different threads are twisted together is as follows: The product of all the counts $\div$ the sum of the products of the combinations of n counts taken $(n-1)$ at a time = the resulting count of the compound structure. From this result the necessary equation can be found by adjusting the terms, as shown, if it should be desirable to find the count of a certain thread to combine with three or more other known counts to obtain a fixed resulting count. It need hardly be said that

the simplest formula should be used in all cases, although the general formula is the foundation of all the other methods.

Occasionally, and principally in examination papers, it is desired to find the average count of the threads when different numbers and different counts—single yarn or multiple-ply—are used in the same warp. As a rule, however, and for obvious reasons, these groups are, in practice, calculated separately. Nevertheless, we show the process so that it may be referred to if desired.

The threads in a warp for a striped pattern are arranged in the following order:

12	threads of	40's
2	,,	8's
10	,,	20's
2	,,	8's.

There are no broken or part patterns. What is the average count of the warp threads?

$$3\begin{cases} 40 \text{ hanks} \times 12 \text{ threads} \div 40\text{'s} = 12 \text{ lb.} \\ 40 \quad ,, \quad \times 10 \quad ,, \quad \div 20\text{'s} = 20 \text{ lb.} \\ 40 \quad ,, \quad \times \ 4 \quad ,, \quad \div \ 8\text{'s} = 20 \text{ lb.} \end{cases}$$

$$\overline{120} \text{ hanks} \quad \overline{26} \text{ threads} \qquad \overline{52 \text{ lb.}}$$

Hence

$$\frac{120 \times 26}{3 \times 52} = 20\text{'s average count.}$$

Another way of finding the average count for the

same or any other pattern is to take any number of hanks of each count in order to have the same length of each, and find the proportionate quantities according to the amount of each used. Thus, if x represents the number of hanks of each, then in the above pattern we should have:

$$\frac{12}{x} = \text{No. of hanks of 40's},$$

$$\frac{10}{x} = \quad ,, \quad ,, \quad 20\text{'s},$$

$$\frac{4}{x} = \quad ,, \quad ,, \quad 8\text{'s}.$$

Perhaps the most convenient number for x is represented by the number of threads in the pattern. In the above case the number is

$$12+2+10+2=26,$$

hence we might use 26 hanks of each:

$$\begin{aligned} \tfrac{12}{26} \text{ of } 26 \text{ hanks} \div 40\text{'s} &= \tfrac{3}{10} \text{ lb.} \\ \tfrac{10}{26} \text{ of } 26 \quad ,, \quad \div 20\text{'s} &= \tfrac{5}{10} \text{ lb.} \\ \tfrac{4}{26} \text{ of } 26 \quad ,, \quad \div 8\text{'s} &= \tfrac{5}{10} \text{ lb.} \\ \hline & 1\tfrac{3}{10} \text{ lb.} \end{aligned}$$

and

$$\frac{26 \text{ hanks}}{1\tfrac{3}{10} \text{ lb.}} = \frac{26 \times 10}{13} = 20\text{'s count.}$$

If there are broken patterns, the proportion of threads must be in terms of the total number of threads in the warp.

CHAPTER V

MULTIPLE-PLY YARNS: SHRINKAGE CONSIDERED

In actual practice no resulting count would be considered correct unless the amount of contraction which took place during the operation of twisting was considered. The conditions to be observed in, and the various phases in connection with, this important part of the subject are outside the scope of this work; they are, however, none the less important, and the reason for their omission in this work will be quite well understood. Perhaps this particular branch will form the subject of a special work; in the meantime, any percentages of take-up, or extra lengths required for a given length of twist, will be taken from the point of view of simplicity, and with the sole object of demonstrating the principles involved in the calculations of the counts of twisted threads. Nevertheless, the relative lengths of the single and twist yarns will be near the actual values obtaining in many ordinary cases. In this respect an exposition of the general formulæ should perhaps

appear first, and numerical examples follow for confirmation.

From the definition of the counts it is advisable, and sometimes necessary, to state the lengths of yarn used in terms of hanks, cuts, leas, or skeins, in either whole or mixed numbers; and it will be understood that in actual practice the lengths used will, in most cases, consist of both. Again, although it is usual in educational work to give the amount of shrinkage in terms of percentage of the length used—*i.e.* the length of the single yarn—in practice it is more common to have concrete quantities of the respective lengths of the single and twisted yarns. Both methods shall be explained.

An extraordinary amount of twist yarn of various kinds is made from the different fibrous yarns, and in many cases a very large number of threads are twisted together. In these cases it is well-nigh impossible to utilise exactly the same length of all the individual yarns which are united by twisting, although in the making of an ideal thread this condition should be approached as near as it is practicable. Moreover, the discussion of such is more concerned with rope and twine making than with compound yarns which are intended for weaving purposes. It is well known, however, that the introduction of motor-tyre fabrics and the like involves the use of compound yarns in which a considerable number of threads are twisted

together, and in which extreme regularity and uniformity are highly desirable. The principles about to be explained should, however, be sufficient to satisfy most cases, especially if one can estimate, or find out by experiment, the average length of the threads compounded.

In the fixed-length systems the process is comparatively simple on account of the reasons already mentioned. For the sake of discriminating between the two distinct systems, we shall adopt different letters for the lengths used. Thus, in the fixed-length system, let

l = the fixed length in yards (see column 6, Table II. p. 17) of the twisted material, or any other length of twist;

L = the length which, when twisted, assumes the twisted length l;

L_1, L_2, L_3, . . . L_n = the lengths of different individual threads of the same or different count;

G_1, G_2, G_3, . . . G_n = the counts of the individual yarns;

R = the resulting count of the compound yarn;

N = the number of threads in combination.

The simplest case is that in which two or more threads are twisted together, and in which the take-up in all the individual threads is the same.

Then we have the following:

$$\frac{L}{l} \times GN, \text{ or } \frac{LGN}{l} = R.$$

Example: Two 5-lb. jute yarns are twisted together; if 12 in. of the twist requires $12\frac{1}{2}$ in. of each of the single yarns, find the resulting count.

$$\frac{12\frac{1}{2} \text{ in.} \times 5 \text{ lb.} \times 2 \text{ threads}}{12 \text{ in. twist}} = \frac{62 \cdot 5}{6}$$
$$= 10\tfrac{5}{12} \text{ lb., or } 10 \cdot 416 \text{ lb.}$$

Two 5-lb. jute threads are twisted together; if the take-up in twisting is 4 per cent, what is the resulting count?

No length need be considered for questions such as this; hence

$$5 \text{ lb.} \times 2 \text{ threads} \times \frac{100}{96} = 10\tfrac{5}{12} \text{ lb., or } 10 \cdot 416 \text{ lb.}$$

General formula:

$$\frac{G \times N \times 100}{(100 - \% \text{ take-up})} = R.$$

This latter is, of course, quite different from the case where the percentage is to be added to the length of the twist yarn. Thus, if one were asked what would be the resulting count if, when two 5-lb. single yarns were twisted together, each single yarn were 4 per cent longer than the twisted thread, we should proceed as under:

$$5 \text{ lb.} \times 2 \text{ threads} \times \frac{104}{100} = 10\tfrac{2}{5} \text{ lb., or } 10{\cdot}4 \text{ lb.}$$

General formula:

$$\frac{G \times N \times (100 + \text{allowance over twisted length})}{100} = R.$$

Of course there is, in reality, very little difference in the two particular cases cited, and no appreciable error could be detected whichever method happened to be used; in other cases, however, the use of the wrong formula instead of the right one might make a difference which is not negligible. Any of the systems mentioned in Table II., p. 17, or similar systems not included in that table, may be treated in a like way.

If all the threads are of different counts, and different lengths of each are required for a given length of twist, then the general formula is

$$\left(\frac{L_1G_1}{l} + \frac{L_2G_2}{l} + \;.\;\;.\;\;.\; + \frac{L_nG_n}{l}\right) = R.$$

Example: Suppose 12 yds. of a compound yarn requires 16 yds. of 4's, 15 yds. of 8's, and 12 yds. of 20's, what is the resulting count of the compound yarn?

$$\left(\frac{16 \times 4}{12} + \frac{15 \times 8}{12} + \frac{12 \times 20}{12}\right) = R.$$

$$\therefore \frac{16 + 30 + 60}{3} = \frac{106}{3} = 35\tfrac{1}{3}\text{'s count,}$$

or

$$5\tfrac{1}{3} + 10 + 20 = 35\tfrac{1}{3}\text{'s.}$$

A different method of calculation from the above is naturally essential for the determination of the twisted counts of those threads which come under any or all of the fixed-weight systems. Commencing with the simplest and the most widely practised method—that is, where two threads only are twisted together, and where the take-up during twisting is the same in both threads, we may assume that two threads of 20's cotton are twisted together, and that 10·3 in. of each single yarn are required for 10 in. of the twist. If these proportions are representative of the bulk, then it is clear that 103 hanks of each yarn, or 206 hanks in all of single 20's, will be required to make 100 hanks of the twist. Hence, keeping in mind the simple rule already mentioned, that any number of hanks divided by the count equals the weight in pounds, or *vice versa*, and, of course, from the definition of the counts, we have

$$\frac{\text{206 hanks of 20's}}{\text{20's}} = \text{actual weight in pounds of the material,}$$

and

$$\frac{\text{100 hanks of twist}}{\text{weight}} = \text{the twist count.}$$

$$\therefore 100 \div \frac{206}{20} = \frac{100 \times 20}{206}$$

$$= 9{\cdot}8\text{'s.}$$

Ten inches of twist yarn is probably the best

length to take, because the calculation is comparatively simple, and the length usually sufficient to obtain a satisfactory average of the take-up. If greater accuracy is required, two or three lengths of 10 in. each might be tested, from the average of which a nearer approach to the actual single and twisted lengths could be obtained.

We are not aware that it is usual, except in the cases cited, and in mills where accurate and scientific methods are followed, to make provision in the spinning department for the take-up in order that the resulting count of the twisted thread may be a whole number. If, however, such provision were necessary, then the procedure, say, in the case of a twisted count equal to 10's, and in which the take-up was equivalent to $\frac{1}{2}$ in. in 10 in., would be as follows: $10\frac{1}{2}$ in. of each yarn for 10 in. of twist equals 105 hanks of each of the single yarns for 100 yds. of twist. Therefore

$$\frac{100}{210} \times G = R = 10\text{'s in this case.}$$

$$\therefore G = \frac{10 \times 210}{100}$$

$$= 21\text{'s single for each yarn.}$$

Somewhat similar allowances for loss of weight will be necessary in gassed yarns, bleached yarns, and the like.

If, for the general formula, we adopt the following :

H_1 = Number of hanks, leas, skeins of No. 1 thread,
H_2 = " " " " " 2 "
H_n = " " " " " n "
h = " " " " the compound yarn,
$G_1, G_2, \ldots G_n$ = the different counts of $H_1, H_2, \ldots H_n$,
W = the weight of $H_1 + H_2, \ldots + H_n$,
R = the resulting count of the twist ;

then

$$\frac{H_1}{G_1} + \frac{H_2}{G_2} + \ldots + \frac{H_n}{G_n} = W;$$

and

$$\frac{h}{W} = R;$$

or

$$h \div \left[\frac{H_1}{G_1} + \frac{H_2}{G_2} + \ldots + \frac{H_n}{G_n}\right] = R.$$

Thus, suppose four threads (30's, 20's, 10's, and 5's) are twisted together, and there are necessary the following lengths :

$$\left.\begin{array}{lll} 12 \text{ in. of} & 30\text{'s} \\ 11 \quad " & 20\text{'s} \\ 10\tfrac{1}{2} \quad " & 10\text{'s} \\ 10 \quad " & 5\text{'s} \end{array}\right\} \begin{array}{l}\text{for 10 in.}\\ \text{of twist.}\end{array}$$

That is,

$$\left.\begin{array}{lll} 120 \text{ hanks of} & 30\text{'s} \\ 110 \quad " & 20\text{'s} \\ 105 \quad " & 10\text{'s} \\ 100 \quad " & 5\text{'s} \end{array}\right\} \begin{array}{l}\text{for 100 hanks}\\ \text{of twist.}\end{array}$$

Then

$$\frac{120}{30}+\frac{110}{20}+\frac{105}{10}+\frac{100}{5}=\text{weight in pounds},$$

or

$$4\ +5\tfrac{1}{2}\ +10\tfrac{1}{2}+\ 20\ =40\text{ lb.}$$

$$\therefore\ \frac{\text{100 hanks of twist}}{\text{40 lb. of material}}=2\tfrac{1}{2}\text{'s twisted count.}$$

If all the threads are of the same count, but different lengths of each, the formula is simplified a little, and obviously becomes

$$\frac{H_1+H_2+\ldots+H_n}{G}=W;$$

and, as before,

$$\frac{h}{W}=R;$$

or, stated in one formula,

$$h\div\left[\frac{H_1+H_2+\ldots+H_n}{G}\right]=R.$$

As already mentioned, it is sometimes necessary in the manufacture of fancy yarns to combine two or more yarns, each of which might differ in fibre from all the others. When the constituent yarns are of different materials, as well as of different lengths, it is usual first to bring all the yarns under the same denomination, and then to proceed as exemplified in the foregoing examples. For instance, suppose that for 10 in. of a fancy knop twist the following were necessary :

1 thread of dark woollen, 160 yds. per ounce Dewsbury count, taking 10 in.				
1	,,	12's black cotton	,,	11 in.
1	,,	10's red worsted	,,	18 in.
1	,,	10's green worsted	,,	18 in.

If the majority of the threads in the warp happened to be woollen, the cotton yarn and the worsted yarns should be brought to the Dewsbury count.

$$12\text{'s cotton} = \frac{12 \times 840,}{16} \text{ or } 12 \times \frac{105}{2} = 630\text{'s Dewsbury.}$$

$$10\text{'s worsted} = \frac{10 \times 560}{16}, \text{ or } 10 \times 35 = 350\text{'s} \quad ,,$$

Make the calculation for 630 yds. of twist, because the 160 yds. per ounce woollen used would be 630 yds. in length, since there is no contraction in this yarn. Then we have

$$\underbrace{\frac{630 \times 10}{160 \times 10}}_{\substack{\text{Woollen}\\\text{thread.}}} + \underbrace{\frac{630 \times 11}{630 \times 10}}_{\substack{\text{Cotton}\\\text{thread.}}} + \underbrace{\frac{630 \times 18}{350 \times 10} + \frac{630 \times 18}{350 \times 10}}_{\substack{\text{Worsted}\\\text{threads.}}} = \text{the weight in ounces of 630 yds. of twist.}$$

$$3{\cdot}94 + 1{\cdot}10 + 3{\cdot}24 + 3{\cdot}24 = 11{\cdot}52 \text{ oz.}$$

$$\therefore \frac{630 \text{ yds. of twist}}{11{\cdot}52 \text{ oz.}} = 54{\cdot}69 \text{ yds. per ounce Dewsbury.}$$

The same combination of four threads may be brought to the worsted count, as under:

$$12\text{'s cotton} = \frac{12 \times 840}{560}, \text{ or } 12 \times \frac{3}{2} = 18\text{'s worsted.}$$

160's Dewsbury $= \frac{160 \times 16}{560}$, or $160 \times \frac{1}{35} = 4 \cdot 57$'s worsted.

In this case take 457 hanks of twist, and also the same length of the 160 yds. per ounce woollen (equal to 4·57's worsted). Then

$$\frac{457 \times 10}{4 \cdot 57\text{'s} \times 10} + \frac{457 \times 11}{18\text{'s} \times 10} + \frac{457 \times 18}{10\text{'s} \times 10} + \frac{457 \times 18}{10\text{'s} \times 10} = \text{weight in pounds.}$$

$$100 + 27 \cdot 92 + 82 \cdot 26 + 82 \cdot 26 = 292 \cdot 44 \text{ lb.}$$

$$\therefore \frac{457 \text{ hanks of twist}}{292 \cdot 44 \text{ lb.}} = \begin{array}{c}1 \cdot 56\text{'s worsted, and practically} \\ 1\text{'s cotton.}\end{array}$$

The reader might check these results by means of Table III., p. 30.

CHAPTER VI

THE PRICE OF TWISTED YARNS AND MIXTURES

A VERY important phase in yarn calculations is that which deals with the price of yarns when two or more threads are twisted together, and when two or more different fibres are used in the composition of a single yarn. As already indicated, such a composition might involve two or more different materials, each in its pure or unadulterated state. On the other hand, some or all of the materials might be in the form of a mixture of two or more different fibres, or two or more different qualities of the same fibre. No absolute instructions can be given as to the exact quantity and price of each individual material, since so much depends upon the result to be achieved in the yarn, and, ultimately, in the fabric, not only as regards appearance, but also in respect of price, durability, and supply.

A common and general rule adopted with regard to the cost of a two-ply or two-fold twist yarn, in which the prices of the two yarns differ, is as

follows: Multiply the highest count by the price of the lowest; add the result to the product of the lowest count and the price of the highest count, and divide this sum by the sum of the counts. Thus, suppose that a 10's yarn at 6d. per pound is twisted with a 12's yarn at 8d. per pound, the cost of the compound yarn, neglecting the take-up in twisting, would be

$$\frac{(12 \times 6) + (10 \times 8)}{12 + 10} = \frac{72 + 80}{22} = \frac{152}{22} = 6\tfrac{10}{11}\text{d. per pound.}$$

Although the above method gives the correct result under the given conditions, the reason for its correctness might not be sufficiently clear. The formula is really an abbreviation of a general rule based upon the definitions and examples which have already been enunciated.

The product of the above two counts is $12 \times 10 = 120$.

Adopting 120 hanks as the number for calculation purposes, we have

$$\frac{120 \text{ hanks}}{10\text{'s count}} = 12 \text{ lb. at 6d.} = 72$$

$$\frac{120 \text{ hanks}}{12\text{'s count}} = 10 \text{ lb. at 8d.} = 80$$

$$22 \text{ lb.} \quad \text{Cost } 152\text{d.}$$

It will thus be seen that the twist yarn is composed

of 12 lb. of 10's and 10 lb. of 12's, or 22 lb. in all; and the full formula is obviously

$$\frac{(\text{Weight in lb. of 10's} \times \text{price per lb.}) + (\text{weight in lb. of 12's} \times \text{price per lb.})}{\text{Weight in lb. of 10's} + \text{weight in lb. of 12's}}$$
$$= \text{price per lb. of the twist.}$$

The complete numerical formula for the above yarns and prices is, consequently,

$$\frac{\frac{12 \times 10}{10} \times 6 + \frac{10 \times 12}{12} \times 8}{\frac{120}{10} + \frac{120}{12}} = \frac{72 + 80}{22} = 6\tfrac{10}{11}\text{d. per lb.}$$

The above formula, after cancellation, is exactly as shown above.

Now, the above method is evidently applicable in every case, no matter how many threads are combined in the twisted product. The product of all the counts might be taken for the number of hanks as demonstrated in the above example; or, if it should happen to be simpler, the least common multiple of the counts could be taken.

The general formula for any number of threads is as under:

Let $a, b, c \ldots n$ = the different counts; p_a, p_b, $p_c \ldots p_n$ = the price per pound of the different counts; and P = the price per pound of the twist. Then

$$\frac{\dfrac{abc\ldots n}{a}p_a+\dfrac{abc\ldots n}{b}p_b+\ldots+\dfrac{abc\ldots n}{n}p_n}{\dfrac{abc\ldots n}{a}+\dfrac{abc\ldots n}{b}+\ldots+\dfrac{abc\ldots n}{n}}=\text{P}.$$

We supplement the formula with a numerical example in which three threads are twisted together, the counts and prices being as below:

1 thread of 5's at 6d. per pound.
1 " 10's " 8d. "
1 " 40's " 9d. "

$$\frac{\dfrac{5\times10\times40}{5}\times6+\dfrac{5\times10\times40}{10}\times8+\dfrac{5\times10\times40}{40}\times9}{\dfrac{5\times10\times40}{5}+\dfrac{5\times10\times40}{10}+\dfrac{5\times10\times40}{40}}=\text{P};$$

i.e. $$\frac{2400+1600+450}{400+200+50}=\frac{4450}{650}=6\tfrac{11}{13}\text{d.}$$

A rather quicker way in this case would be to take the L.C.M. of 5's, 10's, and 40's—that is, 40—instead of the product of the three counts. Then the example becomes

$$\frac{\dfrac{40}{5}\times6+\dfrac{40}{10}\times8+\dfrac{40}{40}\times9}{\dfrac{40}{5}+\dfrac{40}{10}+\dfrac{40}{40}}=\text{P};$$

i.e. $$\frac{48+32+9}{8+4+1}=\frac{89}{13}=6\tfrac{11}{13}\text{d. as before.}$$

An even quicker way to solve such problems, and perhaps the simplest and safest way in all cases, is as under:

40 hanks ÷ 40's	= 1 lb.	at 9d.	= 9
40 „ ÷ 10's	= 4 lb.	„ 8d.	= 32
40 „ ÷ 5's	= 8 lb.	„ 6d.	= 48
	13 lb.	Cost	89d.

and

$\frac{89\text{d.}}{13 \text{ lb. twist}} = 6\frac{11}{13}\text{d.}$ per pound, as in the above two examples.

When a single yarn is to be manufactured to come within a specified price, it is often necessary, especially in the lower grades of the woollen industry, and also in some of the very high grades in the same industry, to effect a mixture of various kinds of fibres in the one case, and a somewhat similar but modified mixture of different fibres, or a mixture of various shades of the same fibre in the other case. This operation is termed "blending" and "mixing" in some industries, and "batching" in other industries, and in certain cases it demands an extensive knowledge of yarn structure, and sometimes of cloth structure, together with good judgment in regard to other matters. The arithmetical methods which are usually exhibited in text-books and in textile literature in general are theoretically sound, but in practice there are other extremely important functions to consider —functions which often render some of the above-mentioned methods impracticable. It must, of course, be admitted at once that, whatever else

obtains, the price of the mixture must come within the specified amount, or otherwise the manufacture of the yarn will result in a loss. On the other hand, there is the equally important condition that the yarn must fulfil the purpose for which it is intended, or the trade will be diverted to some other quarter.

No hard-and-fast rule can be observed with regard to blending, as conditions of manufacture, supply, and price make frequent changes essential; but we think it is only fair to mention the above facts, so that those who have opportunities of undertaking positions which include the process of blending will not look upon this subject as being one which demands simply the arrangement of any arbitrary quantities of various materials to bring the sum total of a mixture to an amount which, when divided by the weight of that mixture, yields a specified price per pound. The fulling or milling quality, if wool, the colour, the length of staple, and other considerations, may all require to be considered; and, when there is such an extraordinary difference between the highest-priced and the lowest-priced wools, and, in a lesser degree, between the same limits in shoddy, mungo, waste, and the various vegetable fibres, it will be understood that there is a considerable number of ways of making up a mixture of three or more varieties to cost a certain price, but the various changes in the prices, due to the qualities of the materials used in the mixture, may

necessitate a corresponding change in the quantities of the different kinds and qualities of the materials.

If, however, two materials of fixed prices are to be blended to yield a certain price for the mixture, and independent of other considerations, then the quantities of each can be easily found. Thus, suppose 1000 lb. of a mixture of the value of 15d. per pound is to be made from wool noils at 18d. per pound and cotton at 6d. per pound.

Let x = the number of pounds of wool; then $1000 - x =$ the number of pounds of cotton. Hence

$$18x + 6(1000 - x) = 15 \times 1000,$$
$$18x + 6000 - 6x = 15000,$$
$$12x = 15000 - 6000;$$
$$\therefore x = \frac{9000}{12},$$
$$= 750 \text{ lb. of wool,}$$

and

$$(1000 \text{ lb.} - 750 \text{ lb.}) = 250 \text{ lb. of cotton};$$

and so on for any other mixture which involves only two kinds.

Whether the full weight of the mixture is given or not, an amount can be assumed for the sake of calculation, for it is evident that the same cost of the mixture would result on paper, no matter whatever number of pounds was taken for the bulk.

If for any reason it were found advisable to obtain

the result without reference to the bulk, then one might proceed as below:

Let $w_1, w_2 \ldots w_n$ = the weights of the various kinds; $p_1, p_2 \ldots p_n$ = the prices of the various kinds; and P = the price per pound of the mixture. Then, if two kinds only are mixed, we have

$$\frac{w_1p_1 + w_2p_2}{w_1 + w_2} = P;$$

or $$w_1p_1 + w_2p_2 = (w_1 + w_2)P,$$

i.e. $$w_1p_1 + w_2p_2 = w_1P + w_2P;$$

hence $$w_1p_1 - w_1P = w_2P - w_2p_2,$$

and $$w_1(p_1 - P) = w_2(P - p_2).$$

$$\therefore \frac{w_1}{w_2} = \frac{P - p_2}{p_1 - P}.$$

Adopting the same values as in the last example, we have

$$\frac{w_1}{w_2} = \frac{15 - 6}{18 - 15}$$
$$= \frac{9}{3}.$$
$$\therefore \frac{w_1}{w_2} = \frac{3}{1};$$

or, 3 lb. of w_1 (wool) to 1 lb. of w_2 (cotton), which proportions are obviously identical with those already found—viz. 750 lb. of wool and 250 lb. of cotton.

The former method is, however, much the simpler, and besides, in most cases in practice, a specified weight of the blend would be fixed.

The numerical example just solved may be represented algebraically as under:

$$18x + 6(1000 - x) = 15 \times 1000,$$
$$p_1x + p_2(1000 - x) = \mathrm{P} \times 1000,$$

whence

$$\mathrm{P} = \frac{p_1x + p_2(1000 - x)}{1000},$$

and, if W = the total weight of the blend (1000 lb. in above case), the last formula becomes

$$\mathrm{P} = \frac{p_1x + p_2(\mathrm{W} - x)}{\mathrm{W}};$$

or

$$p_1x + p_2(\mathrm{W} - x) = \mathrm{PW}.$$
$$\therefore p_1x + p_2\mathrm{W} - p_2x = \mathrm{PW},$$
$$x(p_1 - p_2) = \mathrm{PW} - p_2\mathrm{W},$$
$$x = \frac{\mathrm{W}(\mathrm{P} - p_2)}{p_1 - p_2};$$

from which result it will be seen that a decrease in the price of the higher-priced material p_1 reduces the value of the denominator, and therefore increases the weight x of that material. A decrease in the price of the lower-priced material p_2 will also increase the quantity x of the higher-priced material, because every reduction of one penny in the price of the p_2 material increases the value of the denominator by 1, but at the same time it means an increase of W in the value of the numerator. It will thus be seen that a decrease in the price of either of the materials means

an increase of weight of the more valuable one, and, of course, a decrease in the price of both materials results in an increase of weight of the higher-priced material. The limit is evidently reached when the whole of W is the same value, and equal to price P per lb., in which case there may be any proportions desired of the two kinds of material, since both are equal in price. Raising the price of either or both has the opposite effect.

In the foregoing equation,

$$P = \frac{p_1 x + p_2(W - x)}{W},$$

the letters W and x, as well as the term $(W - x)$, clearly refer to three distinct weights in pounds. In order to prevent any ambiguity in what follows, we propose to make a slight alteration, or rather substitution. Thus, although we shall retain the letter W for the total weight, we shall replace the terms x and $(W - x)$ by the terms $(P - p_2)$ and $(p_1 - P)$ respectively, because we desire that each of these terms shall represent the difference between two prices as well as the weight in pounds. With these substitutions the equation

$$P = \frac{p_1 x + p_2(W - x)}{W}$$

becomes

$$P = \frac{p_1(P - p_2) + p_2(p_1 - P)}{W}.$$

Then

p_1 = the price per pound of the most valuable material,
p_2 = " " least valuable material,
P = " " mixture,
W = the total weight under consideration—*i.e.*,
$W = (P - p_2) + (p_1 - P)$ when weight is considered.

The equation $P = \frac{p_1(P - p_2) + p_2(p_1 - P)}{W}$ may be arranged as under:

$$P = \left\{ \begin{matrix} p_1 \times (P - p_2) \\ + \\ p_2 \times (p_1 - P) \end{matrix} \right\} \div W;$$

or, by leaving out the plus sign, but remembering that the various products within the large brackets must be added, we have

$$P = \left\{ \begin{matrix} p_1 \times (P - p_2) \\ p_2 \times (p_1 - P) \end{matrix} \right\} \div W;$$

hence, using the same prices as in the foregoing numerical example, *i.e.*

$$\begin{aligned} p_1 &= 18\text{d.}, \\ p_2 &= 6\text{d.}, \\ P &= 15\text{d.}, \end{aligned}$$

we have

$$15 = \left\{ \begin{matrix} 18 \times (15 - 6) \\ 6 \times (18 - 15) \end{matrix} \right\} \div W,$$

or

$$15 = \left\{ \begin{matrix} 18 \times 9 \text{ lb.} \\ 6 \times 3 \text{ lb.} \end{matrix} \right\} \div 12 \text{ lb.};$$

i.e.
$$15=\left\{\begin{array}{rl}18\times 9 = & 162 \text{ pence}\\ 6\times 3 = & \underline{18} \quad ,,\\ \overline{12}\text{ lb.} & 180 \text{ pence.}\end{array}\right\}\div 12 \text{ lb.}$$

$$\therefore \frac{180 \text{ pence}}{12 \text{ lb.}}=15 \text{ pence per pound of the mixture.}$$

The above is an arithmetical method, known as "alligation," for finding the price of a mixture of a number of ingredients of different values.

It is not difficult to show that any number of pairs of materials could be treated similarly to that where two kinds at 18d. and 6d. respectively were proportioned to give a mixture at 15d. per pound. Suppose, for example, that a second mixture were required at the value of 15d. per lb. to be made from wool at 22d. per pound and cotton at 7d. per pound. In this case $p_1=22$, and $p_2=7$. Then

$$15=\left\{\begin{array}{l}22\times(15-7)\\ 7\times(22-15)\end{array}\right\}\div \text{W},$$

$$15=\left\{\begin{array}{rl}22\times 8 = & 176 \text{ pence}\\ 7\times 7 = & \underline{49} \quad ,,\\ \overline{15} & 225\end{array}\right\}\div 15 \text{ lb.}$$

$$\therefore \frac{225 \text{ pence}}{15 \text{ lb.}}=15\text{d. per pound of the mixture.}$$

Again, all the four kinds mentioned in the above examples, and at prices 22d., 18d., 7d., and 6d., may be compounded, if desired, to make a mixture at the

same price—*i.e.* 15d. per pound. In all cases, however, it is necessary to group the materials in pairs so that the price of the mixture may be lower than one of each pair and higher than the other in each pair.

The arrangement may be shown as under:

Let p_1 = the material at 22d.
p_2 = „ „ 18d.
p_3 = „ „ 7d.
p_4 = „ „ 6d.
p_1 and p_3 may be grouped together; or
p_1 „ p_4 „ „ „

It is unnecessary at present to show both, so we will group p_1 and p_4 for one pair, and p_2 and p_3 for the other.

$$P = \left\{\begin{array}{l} p_1 \times (P - p_4) \\ p_4 \times (p_1 - P) \\ p_2 \times (P - p_3) \\ p_3 \times (p_2 - P) \end{array}\right\} \div W,$$

numerically as under:

$$15 = \left\{\begin{array}{r} 22 \times (15 - 6) \\ 6 \times (22 - 15) \\ 18 \times (15 - 7) \\ 7 \times (18 - 15) \end{array}\right\} \div (9 + 7 + 8 + 3),$$

$$15 = \left\{\begin{array}{rrl} 22 \times 9 = & 198 & \text{pence} \\ 6 \times 7 = & 42 & ,, \\ 18 \times 8 = & 144 & ,, \\ 7 \times 3 = & 21 & ,, \\ \overline{27} & \overline{405} & \end{array}\right\} \div 27 \text{ lb.};$$

$$\therefore \frac{405 \text{ pence}}{27 \text{ lb.}} = 15\text{d. per pound of the mixture.}$$

There are evidently more ways than one of combining quantities from each of the above four kinds to make a mixture worth 15d. per pound. In all cases, however, and as already indicated, the price of the mixture must, obviously, intervene somewhere between the two values which are grouped together as exemplified above. In other words, if p_1, p_2, p_3, and p_4 be the respective prices of any four materials, and P the price of the mixture, then it is absolutely necessary to know the prices of all five before the linking arrangements can be made. Let us assume, for example, that p_1, p_2, p_3, and p_4 in the order stated represent descending values of the four individual materials, and that the price of P is next in value to p_1, that is, higher than p_2, p_3, and p_4. Then, the only possible ways of linking the prices in threes are as under:

	High Price.	Mixture Price.	Low Price.
1st linking	p_1	P	p_2
2nd ,,	p_1	P	p_3
3rd ,,	p_1	P	p_4

No other method is possible, simply because p_1 is the only ingredient greater in value than P.

If, however, both p_1 and p_2 are higher in price than P, then the following linking arrangements are possible:

	High Price.	Mixture Price.	Low Price.
1st linking	p_1	P	p_3
2nd ,,	p_1	P	p_4
3rd ,,	p_2	P	p_3
4th ,,	p_2	P	p_4

The linking arrangements for any other series of prices could be written down in a similar manner.

We might with advantage combine the five original prices employed in order to demonstrate this statement. In doing this, we shall adopt ultimately the usual method of linking together each pair of prices between which the price of the mixture must be.

Recapitulating the five original prices, we have

$p_1 = 22$d. per pound.
$p_2 = 18$d. ,,
$p_3 = 7$d. ,,
$p_4 = 6$d. ,,
$P = 15$d. ,,

Seeing that two of these materials are each greater in value than P, and each of the remaining two materials of a lesser value than P, we have the following four ways of combining the materials:

1. 22d. and 7d., since 15d. is between.
2. 22d. ,, 6d., ,, ,,
3. 18d. ,, 7d., ,, ,,
4. 18d. ,, 6d., ,, ,,

An excellent way of arranging these graphically,

and as stated above, is that which is invariably adopted, and demonstrated below:

LINKING ARRANGEMENTS

1st. 2nd. 3rd. 4th.

$$15 = \left\{ \begin{array}{c} 22 \\ 18 \\ 7 \\ 6 \end{array} \right\} \div w;$$

or, if the prices were arranged in the order stated in the two simpler cases,

1st. 2nd. 3rd. 4th.

$$15 = \left\{ \begin{array}{c} 18 \\ 6 \\ 22 \\ 7 \end{array} \right\} \div w.$$

Using the upper arrangement, we have

$$\begin{array}{lll}
\text{1st linked pair} & \left\{ \begin{array}{l} 22-15=7 \text{ lb. at } 7\text{d.} = 49 \\ 15-7=8 \text{ ,, ,, } 22\text{d.} = 176 \end{array} \right. \\
\text{2nd ,, ,,} & \left\{ \begin{array}{l} 22-15=7 \text{ ,, ,, } 6\text{d.} = 42 \\ 15-6=9 \text{ ,, ,, } 22\text{d.} = 198 \end{array} \right. \\
\text{3rd ,, ,,} & \left\{ \begin{array}{l} 18-15=3 \text{ ,, ,, } 7\text{d.} = 21 \\ 15-7=8 \text{ ,, ,, } 18\text{d.} = 144 \end{array} \right. \\
\text{4th ,, ,,} & \left\{ \begin{array}{l} 18-15=3 \text{ ,, ,, } 6\text{d.} = 18 \\ 15-6=9 \text{ ,, ,, } 18\text{d.} = 162 \end{array} \right. \\
 & \qquad\quad \overline{54 \text{ lb.}} \qquad\quad \overline{810} \text{ pence,}
\end{array}$$

and $$\frac{810 \text{ pence}}{54 \text{ lb.}} = 15\text{d. per pound.}$$

Now, suppose it were necessary to make a mixture

P value 18d. per pound, and the material at 18d. in the above example replaced by another at 15d. per pound. Then there would be only one of the materials of higher value than the mixture, and the total ways would be as under:

$$18=\begin{Bmatrix} 22 \\ 15 \\ 7 \\ 6 \end{Bmatrix} \div w;$$

(1st. 22, 15; 2nd. 22, 7; 3rd. 22, 6.)

1st bracketed pair	22 − 18 = 4 lb.	at 15d. =	60	
	18 − 15 = 3 „	„ 22d. =	66	
2nd „ „	22 − 18 = 4 „	„ 7d. =	28	
	18 − 7 = 11 „	„ 22d. =	242	
3rd „ „	22 − 18 = 4 „	„ 6d. =	24	
	18 − 6 = 12 „	„ 22d. =	264	
	38 lb.		684	pence,

and $\dfrac{684 \text{ pence}}{38 \text{ lb.}} = 18\text{d. per pound.}$

The number of possible combinations for such mixtures by this method is obtained by a modification of the following well-known mathematical rule:

$${}^{n}C_{r} = \frac{n(n-1)(n-2) \ldots\ldots\ldots\ldots (n-r+1)}{r!}$$

where n is the number of ingredients,
C „ symbol for combinations,
r „ number taken at a time,
and ! „ product of the first r consecutive natural numbers.

We have already seen, however, that it is impossible to take each and every pair of values, and every pair is included in the above formula. The method is, as stated, a modification of the above.

To crystallise our remarks, let us imagine that six different materials are to be blended, and that the price P of the mixture is less than three of the sorts and greater than the remaining three. It will therefore be clear that p_1, p_2, and p_3 cannot be used amongst themselves to make pairs, because each is more valuable than P. It is equally clear that p_4, p_5, and p_6 cannot be paired, because all are individually less valuable than P.

The total number of combinations, practicable and impracticable, is, according to the above formula,

$$^6C_2 = \frac{6 \times 5}{1 \times 2} = 15 \text{ combinations.}$$

But we have already pointed out that there are two groups, three in each group, which cannot yield results. Hence, the correct formula is

$$^6C_2 - [^3C_2 + {}^3C_2] = \text{number of possible combinations,}$$

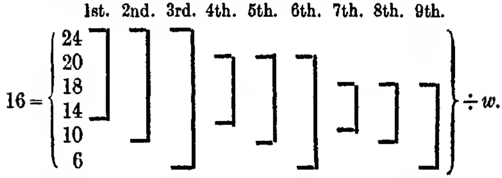
$$\textit{i.e.}\ \frac{6 \times 5}{2} - \left[\frac{3 \times 2}{2} + \frac{3 \times 2}{2}\right] = \qquad \text{do.}$$

or

$$15 - (3 + 3) = 9.$$

This result can be easily demonstrated symbolically or numerically as under. Thus, let the values

of the six sorts be 24, 20, 18, 14, 10, and 6, and the price of the mixture 16d. per pound. Then



Again, the diagrammatic method is simpler than the mathematical one, and, moreover, the problem is half accomplished when the values are linked together as above. Those interested might complete the above example for further confirmation of the principle. They will find for the last stage

$$\frac{1536 \text{ pence}}{96 \text{ lb.}} = 16\text{d. per pound.}$$

We shall see shortly, however, that the above methods do not embrace all the possible ways of combining a number of materials at different prices to obtain a mixture at a fixed price, because one need not be restricted to whole numbers.

Another phase of this subject, and a most important one in the practical process of blending, is that where a mixture should contain a fixed amount of one sort—say, 30, 40, or 50 per cent, or any other percentage of the weight. Suppose, for a further example, it were required to mix or blend the following four materials to obtain a mixture worth 7d. per pound :

Wool at 16d. per pound.
Mungo at 8d. per pound.
Cotton at 6d. per pound.
Hair at 4d. per pound.

When these four materials are arranged in the above diagrammatic fashion, we see that there are four arrangements:

1st. 2nd. 3rd. 4th.

$$7=\left\{\begin{array}{c}12\\8\\6\\4\end{array}\right\}\div w.$$

And if meanwhile we leave out the letter w indicating weight, and letter the four linked pairs by a, b, c, and d, the complete particulars might be indicated as follows:

a. b. c. d.	Lbs. in *a.*	Lbs. in *b.*	Lbs. in *c.*	Lbs. in *d.*	Product of Lbs. and Pence.
					lb. d.
$7=\left\{ 12 \right.$	1	3			4 at 12d. = 48
8			1	3	4 ,, 8d. = 32
6	5		1		6 ,, 6d. = 36
4		5		1	6 ,, 4d. = 24
	6	8	2	4	20 lb. 140

and $$\frac{140 \text{ pence}}{20 \text{ lb.}}=7\text{d. per lb.}$$

With this particular arrangement, and, say, 1000 lb. in the completed mixture, we should have

$$\begin{array}{llr} & & \text{Pence.} \\ 1000\times\frac{4}{20}=200\text{ lb.} & \text{at } 12\text{d.} & =2400 \\ 1000\times\frac{4}{20}=200 \quad ,, & ,, \quad 8\text{d.} & =1600 \\ 1000\times\frac{6}{20}=300 \quad ,, & ,, \quad 6\text{d.} & =1800 \\ 1000\times\frac{6}{20}=300 \quad ,, & ,, \quad 4\text{d.} & =1200 \\ \hline \qquad\quad 1000\text{ lb.} & & 7000\text{ pence.} \end{array}$$

and $$\frac{7000\text{ pence}}{1000\text{ lb.}}=7\text{d. per pound.}$$

In this example there is evidently $\frac{4}{20}\times 100=20$ per cent of the highest-priced material. It is quite possible, however, that in some cases a larger percentage of this sort, if wool, might be necessary for the felting requirements of the cloth into which the spun yarns are to be woven; while in other cases it might be desirable to use a smaller quantity of the wool, or of the highest-priced material, whatever fibre it might be.

The following equation, being an identity, is true for all values:

$$mx+n(100-x)-x(m-n)\equiv 100n,$$

where

x = the percentage of the highest-priced material,
m = the price ,, ,, ,, ,,
n = the price of the mixture.

Suppose, therefore, we keep to the price mentioned

in the last example for the highest-priced material —that is, 12d. per pound—and see what the result would be if 22 per cent of wool were to be added, instead of 20 per cent as found by the diagrammatio method. We have, using the above identical equation,

$$12 \times 22 + 7(100 - 22) - 22(12 - 7) = 100 \times 7.$$

That is, if 22 lb. cost 12d. per pound, then $(100 - 22)$ or 78 lb. must equal in value

$$7(100 - 22) - 22(12 - 7)$$

or $$7 \times 78 - 22 \times 5$$

$$546 - 110 = 436 \text{ pence}$$

since $$700 \text{ pence} - (22 \times 12) \quad = 436 \quad ,,$$

$\therefore$ $$\frac{436 \text{ pence}}{78 \text{ lb.}} = 5\tfrac{23}{39}\text{d. per pound average.}$$

Arranging the remaining three fibres—mungo, cotton, and hair—with values of 8, 6, and 4 pence respectively, in the usual diagrammatic way, we have

$$\begin{array}{c} \text{1st. 2nd.} \\ 5\tfrac{23}{39} = \left\{\begin{array}{c} 8 \\ 6 \\ 4 \end{array}\right\} \div w. \end{array}$$

			Pence.
$8 - 5\frac{23}{39} = 2\frac{16}{39}$	at 4d.	per pound	$= 9\frac{25}{39}$
$5\frac{23}{39} - 4 = 1\frac{23}{39}$	,, 8d.	,,	$= 12\frac{28}{39}$
$6 - 5\frac{23}{39} = \frac{16}{39}$	,, 4d.	,,	$= 1\frac{25}{39}$
$5\frac{23}{39} - 4 = 1\frac{23}{39}$	,, 6d.	,,	$= 9\frac{21}{39}$
6 lb.			$33\frac{21}{39}$ pence,

and $\dfrac{33\frac{21}{39}\text{ pence}}{6\text{ lb.}} = 5\frac{23}{39}$d. per pound as desired.

The above 6 lb. represent, as stated, 78 per cent of the mixture, therefore the 22 per cent of the wool would amount to

$$\frac{6\text{ lb.} \times 22\ \%}{78\ \%} = 1\tfrac{27}{39}\text{ lb.}$$

Multiply all the quantities by 39 to get rid of fractions, and then multiply the weights thus found by the respective prices, and we get the following:

$$\begin{array}{rrlr}
1\frac{27}{39} \times 39 = & 66\text{ lb.} & \text{at } 12\text{d.} = & 792 \\
1\frac{23}{39} \times 39 = & 62 & ,,\ ,,\ \ 8\text{d.} = & 496 \\
1\frac{23}{39} \times 39 = & 62 & ,,\ ,,\ \ 6\text{d.} = & 372 \\
(2\frac{16}{39} + \frac{16}{39}) \times 39 = & 110 & ,,\ ,,\ \ 4\text{d.} = & 440 \\
\hline
 & 300\text{ lb.} & & 2100\text{ pence,}
\end{array}$$

and $\dfrac{2100\text{ pence}}{300\text{ lb.}} = 7$d. per pound.

Then *pro rata* for any size of blend. Thus, if the blend were required to be 1000 lb., we should have

$$\frac{66 \times 1000}{300} = 220\text{ lb. of wool}$$

$$\frac{62 \times 1000}{300} = 206\tfrac{2}{3}\ ,,\ \text{ mungo}$$

$$\frac{62 \times 1000}{300} = 206\tfrac{2}{3}\ ,,\ \text{ cotton}$$

$$\frac{110 \times 1000}{300} = 366\tfrac{2}{3}\ ,,\ \text{ hair}$$

$$1000\text{ lb. in all.}$$

It can be easily proved that there are several more ways of combining the four materials to effect a mixture at 7d. The same or similar principle could be adopted, where of course the conditions desired were capable of solution, if it were advisable to select a particular percentage of any of the three remaining substances.

What probably would be of value are the maximum and minimum quantities of wool to be used in conjunction with the remaining three kinds. These quantities cannot be solved by the above identical equation, as the reader may easily prove by taking an unknown value of x and working out the equation. We may, however, say that the maximum amount of wool will be used when the mixture is almost free from the sorts at 6d. and 8d. And to get the nearest approach to this maximum we may assume that these two sorts are omitted. Then we have two sorts at 12d. and 4d. to make a mixture at 7d. Hence, if $x =$ the quantity of wool, we have

$$\begin{aligned}12x + 4(100 - x) &= 100 \times 7,\\ 12x + 400 - 4x &= 700,\\ 8x &= 300,\\ \therefore x &= 37\tfrac{1}{2}\ \%;\end{aligned}$$

so that anything approaching, but not actually reaching, $37\frac{1}{2}$ per cent of wool could be used, but clearly not more in the case under notice.

The minimum amount of wool to be used would obtain when the sorts at 8d. and 4d. approached zero. (The case of the sorts at 6d. and 4d. at zero is impracticable, because it is obviously impossible to make a mixture at 7d. from individual groups at 12d. and 8d.; but if this condition were overlooked, the calculated result would illustrate the fallacy.)

Hence, by similar calculation, we get

$$12x + 6(100 - x) = 700,$$
$$12x + 600 - 6x = 700,$$
$$6x = 100,$$
$$\therefore\ x = 16\tfrac{2}{3}\ \%\ \text{of wool.}$$

This low percentage may, therefore, be approached, but obviously not reached, if all the four sorts are to be included in the mixture, and this condition was assumed. The maximum and minimum quantities of wool therefore may be taken as $37\frac{1}{2}$ per cent and $16\frac{2}{3}$ per cent, and any percentage between these two extremes may be used, and the calculation made as already demonstrated by the identical equation and by the numerical example which immediately follows the equation.

CHAPTER VII

THE TURNS PER INCH OR TWIST OF YARNS

THE RELATION BETWEEN THE COUNTS OF YARN AND THE "TWIST" PER INCH.—In all yarns, no matter which system of counting they come under, the number of turns per inch, or the technical term "twist," varies according to several circumstances. Thus, in nearly every branch of the textile industry, there are hard, medium, and soft twists, which indicate respectively, and usually, the largest, the medium, and the smallest number of turns per inch for any particular count of yarn. In addition to these three general values there are, in some industries, intermediate degrees of twist between each pair, and also exceptional degrees of twist, some of the latter of which might exceed the value of the so-called hard twist, while others are lower in value than the so-called soft twist.

In spite of the many different degrees of twist which may be imparted to different yarns of any given count, there is not a great difference between the highest

number and the lowest number of turns per inch in such count. In general, the harder twisted yarns are used for warp, and the softer twisted yarns for weft; but there are so many different structures of cloth, such a variety of effects, and so many conditions to be considered, in regard to yarn-making, the lengths and diameters of the constituent fibres, and to the effect which these conditions impart to the finished fabric, that the above general rule cannot be taken as absolute. Moreover, there are large quantities of yarn which are used for other purposes than weaving.

It is impossible to deal with all the phases connected with twist in a work of this kind, and obviously impossible to supply a rule which is applicable to all the different degrees of twist. Rules which are used in practice are not of general application; the various rules are applied within reasonable limits of counts, for a rule which is suitable for fine or high counts may be quite impracticable for low counts. Nevertheless, the fundamental principles which govern the amount of twist for all sizes of yarn of a standard make, and under either the "fixed-weight" system or the "fixed-length" system of counting, can be demonstrated mathematically, and the values found in this way can be, and are, applied with satisfactory results in practice.

It is not our intention to deal here with the question

of absolute diameters of yarns,[1] but, in connection with the application of twist, it is essential at least to say a few words with regard to the relation between the diameters of the various counts and the number of turns per inch.

In the opening chapter of this work it was stated that the number or count of a yarn is either directly or inversely proportional to the sectional area of the yarn according to whether that count is under the "fixed-length" system or the "fixed-weight" system of counting. In both cases the thickness or diameter of the yarn is a function of the sectional area of the yarn. A rough approximation of the relation between the diameters and the sectional areas can be demonstrated as follows:

Four equal squares, each with sides equal to x, appear in Fig. 2. In each square is a circle y,

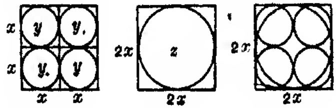


FIG. 2. FIG. 3. FIG. 4.

obviously with a diameter equal to a side of the square, the four circles representing the sectional areas of four threads. In Fig. 3 a square is drawn

[1] See the Author's work on "The Diameters of Yarns and the Structure of Fabrics" (*The Textile Mercury*).

with sides equal to $2x$, and a circle z with a diameter of $2x$ inserted. All the five circles are drawn in a similar square (side $=2x$) in Fig. 4, from which it is evident that whereas four threads, each with diameter equal to x, can be enclosed within the square, only one thread with diameter equal to $2x$ can be enclosed in the same square.

The exact relations between the two different squares and the circles enclosed within them are as follows :

$$\text{Area of square } x = x^2,$$

$$\text{Area of circle } y = \frac{\pi}{4}x^2 \text{ or } 0{\cdot}7854 \times x^2,$$

$$\text{Area of square } 2x = (2x)^2 = 4x^2,$$

$$\text{Area of circle } z = \frac{\pi}{4}(2x)^2 \text{ or } 0{\cdot}7854 \times 4x^2;$$

$$\therefore \frac{\text{Small square}}{\text{Large square}} : \frac{\text{Small circle}}{\text{Large circle}} = \frac{x^2}{(2x)^2} : \frac{0{\cdot}7854x^2}{0{\cdot}7854(2x)^2}$$

i.e.
$$\frac{\text{Small circle}}{\text{Large circle}} = \frac{1}{4}.$$

This is, of course, a roundabout way of proving the relation, but it is a convincing one. Since the relation between squares and circles is constant, and since the square on the whole line, $(2x)$, is four times the square on half the line, (x), as demonstrated graphically in Figs. 3 and 2, it follows that the areas are as 4 to 1, and that the areas of circles with diameters equal to the sides of the two squares are also

4 to 1. Or, if the small circle is mentioned first, the ratio is 1 to 4.

Now we may proceed to demonstrate the principle upon which the degree of twist is imparted to yarns of different sizes. In the first case, we shall consider the amount of twist with reference to the sectional areas of the threads, because these are independent of the methods of counting.

In the upper part of Fig. 5 there are five perfect squares of different sizes, lettered A, B, C, D, and E; immediately below these squares there are five circles lettered similarly, and the diameters of these five circles are identical with the sides of the five different squares, as is shown clearly by the five spaces enclosed between the six long vertical lines. Below these circles, and in the same five spaces, are several zigzag lines which are intended to represent diagrammatically the twists or convolutions of a single fibre in each of the spaces, and, in the meantime, it is assumed that each fibre forms a spiral or helix, of which each and every part appears on the outer layer of the thread. (It is well known that no fibre actually follows this particular configuration.)

All the configuration lines in Fig. 5 are at right angles to each other, the solid lines representing the fibres on the upper surfaces of the threads, and the dotted lines representing the same fibres on the lower surfaces of the threads, so that the angle of twist

represented is one of 45°. The angle shown is more acute than what obtains as a rule in practice, and is

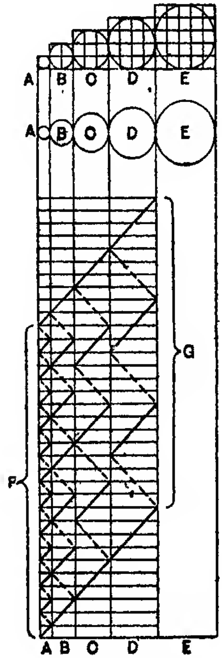


Fig. 5.

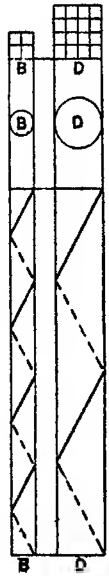


Fig. 6.

adopted here solely for convenience of demonstration. The lines in Fig. 6 are inclined at an angle of 60°, and this is even less than that of the fibres in many yarns.

The upper solid inclined line in Fig. 5 which extends across four of the spaces, A, B, C, and D, is intended to demonstrate the upper half of each twist of each of the four threads, while the lower solid inclined line, which is also continuous through spaces A, B, C, and D, is intended to show the nearest place when all the four threads start under the same condition. The brackets F and G consequently represent a complete cycle, from which it will be observed that in the same length of yarn, F or G, the thread A has 12 turns or twists, the thread B has 6, the thread C has 4, and the thread D has 3. The twists for thread E have been purposely omitted in order to minimise the length of the illustration.

Collecting the information on the drawing, and keeping in mind the fact that the areas of squares are proportional to circles, the diameters of which are equal to the sides of the squares, we have the following table:

TABLE VI

Thread.	Number of Squares or Units in the Sectional Area.	Diameter of Square and Circle.	No. of Twists in Length.	Product of Diameter and Twist.
A	1	$\sqrt{1}$ or 1	12	12
B	4	$\sqrt{4}$ „ 2	6	12
C	9	$\sqrt{9}$ „ 3	4	12
D	16	$\sqrt{16}$ „ 4	3	12
E	25	$\sqrt{25}$ „ 5	$2\frac{2}{5}$	12

The third column in the table shows that the diameters of the five yarns are proportional to the square roots of the corresponding sectional areas, and it may be taken for granted that all other diameters of threads come under the same rule. On the other hand, the fourth column shows distinctly that the number of turns or twists per given length decreases as the diameters and the sectional areas increase, while the last column shows that the product of the diameter and the twist per given length is a constant quantity; in this particular case it is 12. Thus, in the first two threads we have:

$$A\text{; diameter} \times \text{twist} = 1 \times 12.$$

$$B\text{; diameter} \times \text{twist} = 2 \times 6.$$

Now, $$1 : 6 = 2 : 12.$$

$$\therefore A : 6 = B : 12.$$

or $$\frac{A}{6} = \frac{B}{12}.$$

Hence $$\frac{A}{B} = \frac{6}{12}$$

—that is, the turns or twists per inch are inversely proportional to the diameters of the yarns, and, since it has already been shown that the diameters of yarns are proportional to the square root of the sectional areas, it follows that the turns or twists per inch are inversely proportional to the sectional areas of the yarns.

It was demonstrated in connection with Fig. 1,

and stated immediately before this figure, that, in the "fixed-length" system of counting, the count of the yarn was directly proportional to the square root of the sectional area, whereas in the "fixed-weight" system of counting, the count of the yarn was inversely proportional to the sectional area. Since in the "fixed-weight" system the count of the yarn increases as the sectional area decreases, and since the twist, or the number of turns per inch, increases as the sectional area decreases in the manner already demonstrated, it follows that the twist, or the number of turns per inch, of all yarns in this system of counting is directly proportional to the square root of the count of the yarn. It is evident, however, that this proportionality assumes that the angle of the twist is the same in all the different counts for one particular type of structure, and that some standard twist for a certain, and preferably largely used, count must be chosen before the twist for the other counts can be calculated. When once this standard is fixed, it is possible to calculate all the others from it.

In order to demonstrate this particular method of determining the degree of twist, let it be assumed that in a certain kind of 16's cotton yarn the twists per inch are 15. Then, if it is desired to obtain the same angle of twist in a 9's cotton, which, being thicker than 16's, should have fewer turns per inch, we have

$$\sqrt{16} : \sqrt{9} = 15 : x;$$

or
$$4 : 3 = 15 : x.$$
$$4x = 15 \times 3.$$

Hence
$$x = \frac{15 \times 3}{4} = 11\tfrac{1}{4} \text{ turns per inch in 9's.}$$

These particular counts of yarn have been chosen because their numbers represent perfect squares, and consequently their roots are easily extracted. If, however, the extraction of the roots of the counts under consideration involves decimal fractions, it is often the easier plan to square all the terms in the equation. Thus, suppose 12's cotton has 13 turns per inch, how many turns should 24's cotton have? The 24's, being finer than 12's, will have more twist than the latter; therefore

$$\sqrt{12} : \sqrt{24} = 13 : x,$$

which becomes, after squaring each term,

$$12 : 24 = 13^2 : x^2,$$
$$12x^2 = 13^2 \times 24,$$
$$x^2 = \frac{13 \times 13 \times 24}{12};$$
$$\therefore x^2 = 338,$$

or
$$x = 18{\cdot}4 \text{ turns per inch of 24's.}$$

A more general plan in practice is to find a constant number which, when multiplied by the square root

of the count, will give the required twist for all yarns in any particular range. For example, in the first of the above examples, where cotton yarns of 16's and 9's were considered, it is evident from the amount of twist found that the same result would have been obtained if the square roots of the two counts had been multiplied by a constant of 3·75. Thus

$$\sqrt{\text{Count}} \times \text{constant number} = \text{twist};$$
$$\therefore \sqrt{16} \times 3{\cdot}75 = 15 \text{ turns per inch in 16's},$$
$$\sqrt{9} \times 3{\cdot}75 = 11\tfrac{1}{4} \quad \text{,,} \quad \text{,,} \quad \text{9's}.$$

From what has been said about hard, medium, soft, and exceptional twists, it will be clear that the above-mentioned constant, 3·75, will be applicable only to one degree of twist for all counts, and that different constants will be utilised for the different degrees of twist. These constant numbers vary in different branches of the trade. In general their values range between 4 and 1·5, although for certain classes of yarn the constant may exceed 5, while in other cases it is below 1·5. It will, of course, be understood that no definite values can be given unless some specific yarns are considered, and even in this case it is necessary to know whether the yarn is to be starched or woven dry, because yarns for starched warps have invariably less twist than the same count of yarn which is to be woven dry.

Under certain conditions the above methods are

applicable to all yarns which come under the "fixed-weight" system of counting; but in practice it will be found that for many reasons, including those already enumerated, these general rules are not strictly observed.

Since the finest or smallest yarns contain the most twist, it need hardly be said that, in regard to those yarns which are reckoned by the group in the "fixed-length" system, the amount of twist is not directly proportional to the square root of the count. In all such yarns the diameter is still proportional to the square root of the sectional area, and the counts are therefore proportional to the sectional area. Thus if we refer again to Table VI., the second column might be taken to represent not only the sectional area, but also the count of yarn, and hence the third column shows that the diameters are directly proportional to the square root of the count of the yarn. Moreover, the fourth column indicates that the number of turns decreases as the count in the second column increases. It can be proved, in a similar way as before in connection with the "fixed-weight" system, that in the "fixed-length" system of counting the twist is inversely proportional to the square root of the count of the yarn. There are also the same fluctuations in regard to the degree of twist for various reasons, as in those pointed out in regard to the "fixed-weight" system, and similar constants to those mentioned

are used extensively for the yarns in the "fixed-length" groups.

To demonstrate the general method of finding the twist for a certain count, having been given the correct standard twist for a different count, we might assume that in respect to the "fixed-length" system a count represented by 4's has 6 turns per inch, and then find what number of turns should be given to a higher count and thicker yarn, say 16's. Then

$$\sqrt{16} : \sqrt{4} = 6 : x.$$
$$4 : 2 = 6 : x.$$
$$4x = 6 \times 2.$$
$$x = \frac{6 \times 2}{4} = 3 \text{ turns per inch for 16's.}$$

And again, if the count is represented by a number which is not a perfect square, we may say: If an 8's yarn has 4 turns per inch, how many turns should there be in a 14's yarn?

$$\sqrt{14} : \sqrt{8} = 4 : x;$$

or

$$14 : 8 = 4^2 : x^2.$$
$$14x^2 = 4^2 \times 8;$$
$$x^2 = \frac{4 \times 4 \times 8}{14}.$$
$$\therefore x^2 = \frac{64}{7} = 9{\cdot}14.$$
$$x = 3{\cdot}02, \text{ or } 3{\cdot}02 \text{ turns per inch for 14's.}$$

Now, since the twist is inversely proportional to

the square root of the count in these yarns of the "fixed-length" system, we might obtain the constant numbers for these examples. Thus, in the first example we have

$$\frac{1}{\sqrt{4}} \times \text{constant} = 6 \text{ turns per inch.}$$

$$\therefore \text{Constant} = 6 \times \sqrt{4} = 12;$$

and

$$\frac{1}{\sqrt{16}} \times \text{constant} = 3.$$

$$\therefore \text{Constant} = 3 \times \sqrt{16} = 12.$$

Hence both yarns are twisted in the same particular ratio.

In the second example we have

$$\frac{1}{\sqrt{8}} \times \text{constant} = 4.$$

$$\therefore \text{Constant} = 4 \times \sqrt{8} = 4 \times 2{\cdot}83 = 11{\cdot}32;$$

and finally

$$\frac{1}{\sqrt{14}} \times \text{constant} = 3{\cdot}02.$$

$$\therefore \text{Constant} = 3{\cdot}02 \times \sqrt{14} = 3{\cdot}02 \times 3{\cdot}74 = 11{\cdot}29.$$

The constant is really the same as in the first part, and the difference lost between 11·32 and 11·29 is due to the neglect of the third place of decimals.

It will be seen from these examples that when once a satisfactory constant is obtained for any particular group of yarns under the "fixed-length" systems—there will be several groups, as already mentioned—

it is only necessary to divide this constant by the square root of the count, because it is obvious that the equation for the above examples,

$$\frac{1}{\sqrt{\text{Count}}} \times \text{constant} = \text{twist},$$

may be written

$$\frac{\text{Constant}}{\sqrt{\text{Count}}} = \text{twist}.$$

CHAPTER VIII

THE ANGLE OF TWIST

The apparent angle of twist is obviously that which is displayed by the fibres on the surface of the thread. In the process of spinning, however, and especially with long fibres, it is doubtful if any single fibre remains on the outside for its full length. And even for the short fibres, say cotton, it is easy to see that if any fibre should occupy this extreme outside position for its full length, it would require but little friction to remove such fibre from the bulk. The longer the fibres and the thinner the yarn, the more chances there are for each individual fibre to occupy the maximum number of positions between the centre of the yarn and its periphery, and to be bound securely with the component fibres; hence our former remark regarding the degree of twist with different lengths of fibres. And since the yarns are, in general, considered to be approximately of uniform density—although they are scarcely ever so—it might with a fair degree of accuracy be assumed that the mean

diameter of the yarn with regard to the twist is that which embraces one-half of its sectional area, or one-half of the total number of fibres.

Hence, if D = the actual diameter of the yarn,

and d = the mean or twist diameter,

then $D^2\frac{\pi}{4}$ = the sectional area of the yarn,

and $d^2\frac{\pi}{4}$ = half the sectional area of the yarn.

$$\therefore\ d^2\frac{\pi}{4}=\frac{D^2\pi}{4\times 2}, \text{ or } d^2=\frac{D^2}{2};$$

i.e.
$$d=\frac{D}{\sqrt{2}}, \text{ or } d=\frac{D}{1\cdot 4142}$$

for all yarns with uniform density; in other words, the mean or twist diameter of the yarn is approximately $\frac{7}{10}$ths of the actual diameter of the yarn.

We are quite well aware that spinners can, and some do, achieve the correct number of turns without resorting to any but the most elementary arithmetical rules. Nevertheless, if the above remarks—which are not antagonistic but supplementary to the spinner's work—regarding the degree of twist in relation to the length and diameter of any type of fibre can be proved to be proportional, or nearly so, a thorough study of the principles involved might have far-reaching effects, not only on the spinning production of yarn from fibres of different diameters and lengths,* but also upon the actual cost of production of any particular

type of yarn which is made from such fibres. It might, indeed, be the nucleus of an extensive scheme of scientific and experimental research.

To return to the discussion of the mean diameter in reference to twist, it is evident that if once the

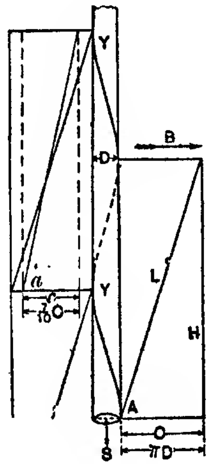


FIG. 7.

actual "working" diameter of a yarn could be found, and assuming that the foregoing remarks are true, we should have

$$\frac{7}{10}D = d, \text{ the twist diameter.}$$

If a yarn Y of diameter D, Fig. 7, be rolled in the

direction of the arrow B, the periphery of the section S would trace out the path C, which is clearly equal in length to the circumference of the outside of the yarn, or πD. At the same time, one complete turn or twist of the fibre on the outside of the yarn would trace out the line L, while the line H would represent the actual length of yarn for one complete twist. The angle A, formed by the two lines L and C, indicates the apparent angle of twist (see the central part of the dotted half of twist), that seen on the outer surface of the yarn. Consequently, this angle A may be taken to indicate the direction of the twist of the fibres when the latter appear on the external part of the thread. Therefore, neglecting positive or negative directions in all cases, we have

$$\frac{H}{C} = \tan A.$$

Now,

$$H = \frac{l}{t} = \frac{\text{length of yarn under consideration}}{\text{number of turns or twist in that length}};$$

but, since it is usual to consider the twist as so many turns per inch, l becomes equal to 1, and, therefore,

$$\text{Tan } A = \frac{1}{Ct}.$$

Finally, since the twist diameter is to be taken as equal to $\frac{7}{10}$D, the actual circumference should be $\frac{7}{10}$C, or, say, c.

$$\therefore \text{Tan } a = \frac{1}{ct}.$$

Without the above substitutions, the latter formula is equal to

$$\text{Tan } a = \frac{10l}{7D\pi t}.$$

Numerical example: A single strand of yarn is 5 in. long, $\frac{1}{16}$ in. diameter, and contains 12 turns in the length. Find

1. The actual angle formed by the fibres, assuming the twist diameter to be $\frac{7}{10}$ths of the real diameter; and
2. The apparent angle of the twist as seen on the outside of the yarn.

In the first case,

$$c = \tfrac{7}{10} \times \tfrac{1}{16} \times 3{\cdot}1416;$$

and
$$t = \frac{12}{5}.$$

$$\therefore \text{ Since } \tan a = \frac{1}{ct},$$

$$\text{Tan } a = \frac{10 \times 16 \times 5}{7 \times 3{\cdot}1416 \times 12},$$

$$\text{Tan } a = 3{\cdot}031.$$

$$\therefore \text{ Angle } a = 71\tfrac{3}{4}^\circ \text{ approx.}$$

In the second case,

$$\text{Tan A} = \frac{1}{Ct} = \frac{16 \times 5}{3{\cdot}1416 \times 12} = 2{\cdot}121.$$

$$\therefore \text{ Angle A} = 64\tfrac{3}{4}^\circ \text{ approx.}$$

In other words, the average inclination of the

fibres is $71\frac{3}{4}°$, whereas the angle presented to the eye, which angle is obviously always on the outside of the yarn, is one of $64\frac{3}{4}°$.

For some special purpose it may be desirable—if not at present, at some future period—to construct a yarn with a definite inclination on the outer surface. The desirability may be said to exist at present in connection with the twisting of multiple yarns, and in some degree in regard to single yarns.

The tangents of the apparent angle A and the average angle a may be represented by the diagram in Fig. 8, in which

$$\text{Tan } A = \frac{H}{C},$$

and

$$\text{Tan } a = \frac{H}{c}.$$

Hence,

$$\frac{\text{Tan } A}{\text{Tan } a} = \frac{\frac{H}{C}}{\frac{H}{c}} = \frac{c}{C} = \frac{7}{10}.$$

$$\therefore \text{Tan } A = \frac{7 \tan a}{10}.$$

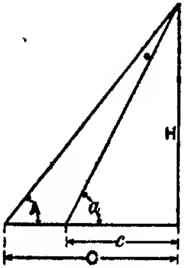


FIG. 8.

Suppose, therefore, that the inclination of the fibres on the outer surface of the yarn should be 60°. Then

$$\text{Tan } 60° = \frac{7 \tan a}{10}.$$

$$\therefore \text{Tan } a = \frac{10 \tan 60}{7} = \frac{10 \times 1{\cdot}7321}{7} = 2{\cdot}474.$$

$$\therefore \text{Angle } a = 68^\circ \text{ approx.}$$

It now remains to find the number of turns per inch, or the twist, on the yarn to produce, say, an apparent inclination of 60°. To do this, we must know the diameter of the yarn, and for this purpose we shall adopt the values deduced by the late Mr. T. R. Ashenhurst, or the empirical equivalent for obtaining these values.[1] That is, deducting 9 per cent, or approximately $\frac{1}{11}$th, from the square root of the yards per pound, and taking a 16's cotton yarn, we obtain the reciprocal of the diameter; thus

$$\sqrt{16 \times 840} - \tfrac{1}{11}\sqrt{16 \times 840} = \text{approx. } 105.$$

$$\therefore \text{Diameter of 16's cotton} = \tfrac{1}{105} \text{ of an inch.}$$

$$\text{Tan A} = \frac{l}{Ct} = \frac{l}{D\pi t}.$$

Let $l = 1$, then

$$\text{Tan } 60 = \frac{1}{\frac{1}{105}\pi t},$$

$$1{\cdot}7321 = \frac{105}{3{\cdot}1416t}.$$

$$\therefore t = \frac{105}{3{\cdot}1416 \times 1{\cdot}7321} = 19{\cdot}3 \text{ turns per inch.}$$

Now this is an excessive number of turns per inch, for since

[1] For working diameters of yarns see the Author's work on *The Diameters of Yarns and the Structure of Fabrics.*

$$\sqrt{\text{Count}} \times \text{constant} = \text{twist},$$

$$\text{Constant} = \frac{\text{twist}}{\sqrt{\text{count}}} = \frac{19{\cdot}3}{\sqrt{16}}$$

$$= \frac{19{\cdot}3}{4} = 4{\cdot}8 \text{ for } 60^\circ \text{ angle.}$$

The constant for warp yarn in cotton is, say, 3·75, hence the turns per inch for 16's warp, or 16's twist as it is often called, is

$$\sqrt{16} \times 3{\cdot}75 = 15 \text{ turns per inch.}$$

Substituting this value for t in the equation immediately above the result of 19·3 turns per inch, we have

$$15 = \frac{105}{3{\cdot}1416 \tan A}.$$

$$\text{Tan } A = \frac{105}{3{\cdot}1416 \times 15} = 2{\cdot}2281.$$

$$\therefore \text{Angle } A = \text{between } 65^\circ \text{ and } 66^\circ.$$

To obtain this angle on the surface of the yarn, and again assuming that the actual twist angle is $\frac{7}{10}$ths of the apparent angle, we have

$$\frac{\text{Tan } a}{\text{Tan } A} = \frac{C}{c} = \frac{10}{7}.$$

Hence

$$\text{Tan } a = \frac{10 \tan A}{7} = \frac{10 \times 2{\cdot}2281}{7} = 3{\cdot}183.$$

$$\therefore \text{Angle } a = 72\tfrac{1}{2}^\circ.$$

In short, it would appear that if, for any calculated

or observed reason, the apparent angle, that presented to the eye of the observer, should be fixed, the twist for the yarn can be found as under if the stated diameter of the yarn can be accepted as correct:

$$\text{Twist} = \frac{1}{\text{diameter} \times \pi \times \tan\ A},$$

which, numerically, would appear as follows:

$$\text{Twist} = \frac{\text{reciprocal of diameter}}{3{\cdot}1416 \times \text{tan of observed angle A}}.$$

It is well known that when two or more threads are twisted together, the outside fibres alone of each thread touch the remaining threads, and the direction followed by the individual fibres, or the angle formed with respect to the length of the compound yarn, can, from observation, refer only to the outside fibres of each single thread. The average direction or the average angle will be different from the observed one, and it becomes a question as to which angle should be adopted when one desires that the fibres of the individual yarns should point in a certain direction. Consider the subject from the following point of view:

Let A = the apparent angle,
a = the average or actual angle,
b = the difference between A and a.

In the numerical examples just demonstrated, A was found to be 66°. and a eoualled $72\frac{1}{4}$°.

$$\text{Tan } 66° = 2{\cdot}2460,$$
$$\text{Tan } 72\tfrac{1}{2}° = 3{\cdot}1716;$$

and drawing these to scale, as in Fig. 9, we see the apparent and average directions followed by the fibres in the first diagram and separated in the second and third diagrams. One quarter of the first diagram

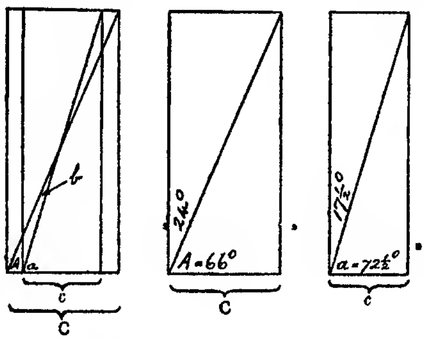


Fig. 9.

is reproduced on a larger scale in Fig. 10. Since angle a = angle A + angle b,

$$\begin{aligned}\text{Angle } b &= \text{angle } (a - \text{A}) \\ &= 72\tfrac{1}{2}° - 66° \\ &= 6\tfrac{1}{2}°.\end{aligned}$$

Suppose that for some particular reason two threads are to be twisted together, and that the constituent fibres should be parallel to the length of the compound yarn. To achieve this condition

the single yarns must be twisted in the reverse direction to the original one (a general practice), but the amount of such twist will depend upon whether one wishes to consider the average angle a or the apparent angle A. Thus, consulting the second and third diagrams in Fig. 9,

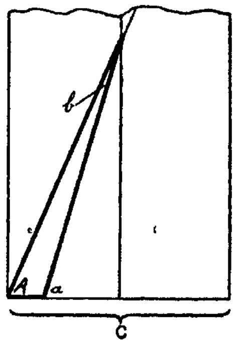


Fig. 10.

For angle a the rotation must equal $17\frac{1}{2}°$,
" A " " " $24°$.

It is impossible to distinguish the direction followed by those parts of the fibres which are embedded in the yarn; those parts on the outside only are visible, but still the calculation may be made with regard to the average angle a formed, since, if what has been stated is correct, the solution of one angle a assures the solution of the other angle A. Moreover, if future

calculations are made with regard to the working diameter of the yarn, it will probably be more convenient to make all calculations with reference to the working diameter, because in cloth structure this diameter plays a more important part than does the theoretical diameter. When the yarns are intended for twine, cords, and the like, the article is the finished product, and hence the observed angle in the cord or twine is undisturbed.

The difference between the average observed diameter and the average working diameter of twisted or compound yarns will, in general, be much less than the difference between the theoretical diameter and the working diameter of a single thread. The relation between the two conditions of the single yarn was taken as 1 to 0·7. It is doubtful whether any alteration should be made in regard to the condition of the working diameter of the single yarn and the condition of the same yarn when used in conjunction with one or more yarns in the formation of a compound yarn.

Although there is a certain degree of tension imparted to the constituent threads of a compound yarn during the operation of twisting, and which naturally decreases slightly the diameter, the untwisting of each single thread during the same operation increases slightly the diameter of each individual thread. Hence we shall assume that no decrease in

the diameter of the single threads takes place during the formation of the compound yarn.

It has been assumed that the diameter of 16's cotton is $\frac{1}{105}$ of an inch, that the average angle a is $72\frac{1}{2}°$, and that no further decrease takes place in the individual threads which are twisted together. It has also been shown that the single yarn must be untwisted through $17\frac{1}{2}°$ to place the fibres parallel to the length of the compound yarn.

One little difficulty arises immediately when we assume that the diameter of 16's cotton is $\frac{1}{105}$ of an inch. We have already stated that this value is obtained from the usual method of finding the reciprocal of the diameter, and as follows:

$$\sqrt{16\text{'s} \times 840} - \tfrac{1}{11}\sqrt{16\text{'s} \times 840}$$
$$= \text{the reciprocal of the diameter} = 105.$$
$$\therefore \text{ The diameter of 16's cotton} = \tfrac{1}{105} \text{ of an inch.}$$

When a constant is used to obtain the same value of the diameter, it takes the form of $\frac{1}{26{\cdot}25\sqrt{\text{count}}}$.

Thus

$$\frac{1}{26{\cdot}25\sqrt{16}} = \frac{1}{26{\cdot}25 \times 4} = \frac{1}{105}.$$

Now, this formula gives the apparent diameter of cotton yarns; a much nearer value for the absolute working diameter is $\frac{1}{32\sqrt{\text{count}}}$. It is desirable to

consider this difference, because it has already been stated that in weaving, the diameter of the thread is almost invariably reduced, whereas in a cord no such subsequent compression is applied. Consequently, in the manufacture of a 2-ply yarn which has not to undergo any compressive stresses, the apparent diameter of the yarn may be used provided that we agree that no change in the diameter of the single yarn takes place during the twisting of the constituent threads. For such work we should use the formula $\frac{1}{26{\cdot}25\sqrt{\text{count}}}$ to obtain the diameter, and hence the diameter of 16's cotton would be $\frac{1}{105}$ of an inch.

The outside fibres in our specific case made an angle of 66°, so that the yarn must rotate through 24° during the operation of twisting if these outside fibres are to be placed parallel to the length of the compound thread.

When, in the conversion of a rove to a thread in the spinning process, the fibres of such yarn have assumed a certain angular direction with respect to the longitudinal direction of the thread, it will be seen that this angular direction can be calculated or observed. And when this yarn is twisted with another yarn of the same kind and count, and with the same amount of twist, the ultimate direction followed by the fibres will obviously depend upon the amount of twist which is imparted to the compound yarn.

If the twist be reversed to that of the single yarn, and the same number of turns per inch be given to the compound yarn as each single yarn originally possessed, all the twists would be taken out of the two individual threads, and each being reduced, as it were, to the sliver state, the fibres would point in the direction followed by the individual threads. If, on the other hand, the compound yarn receives half the amount of twist that was given to the individual yarns, but in the reverse direction, the fibres would be parallel to the direction of the compound yarn. The number of turns in the compound yarn may exceed the number of turns in each of the single yarns, in which case all the original twist in the single yarns would first be removed, and then twist imparted to each single yarn in the opposite direction to that of the original twist and in the same direction as that imparted to the compound yarn. There are evidently considerable variations from the very soft twisted singles and compounds to the very hard ones, and the foregoing remarks and calculations have been introdnced to illustrate some definite course to pursue under certain circumstances.

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